Model-based RL & Decision-Aware Model Learning

(INF8250AE: Introduction to Reinforcement Learning)

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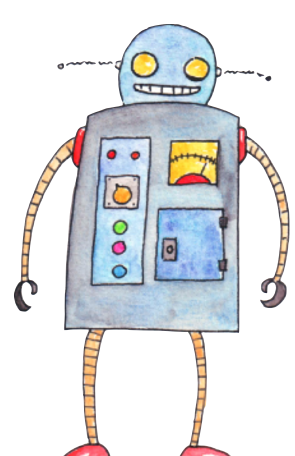

POLYTECHNIQUE MONTRÉAL

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Adaptive Agents Lab (Adage)

Mila

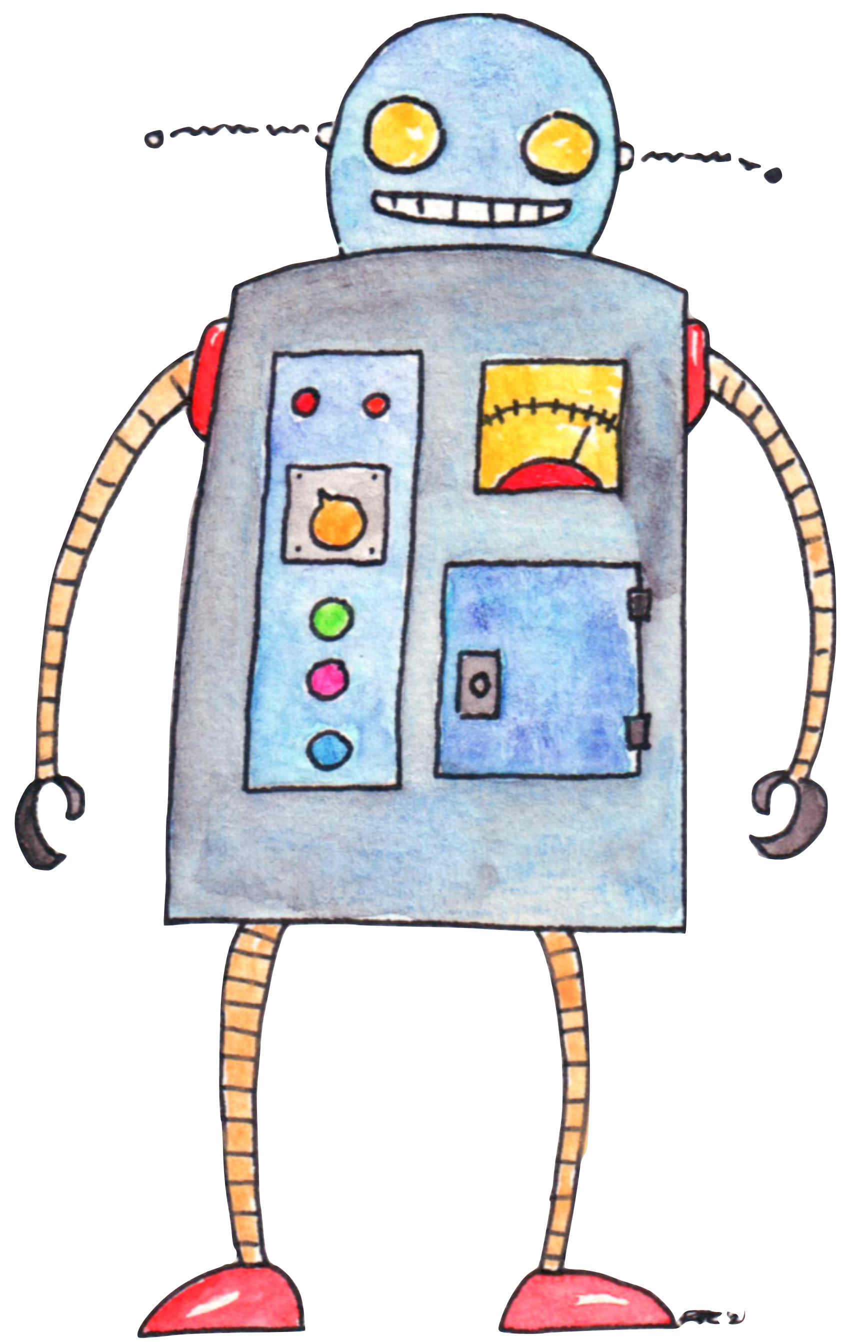
Goal

We study how an RL agent can build a (world) model of its environment and use it to help with its learning. We learn various ways that the agent can learn its model.

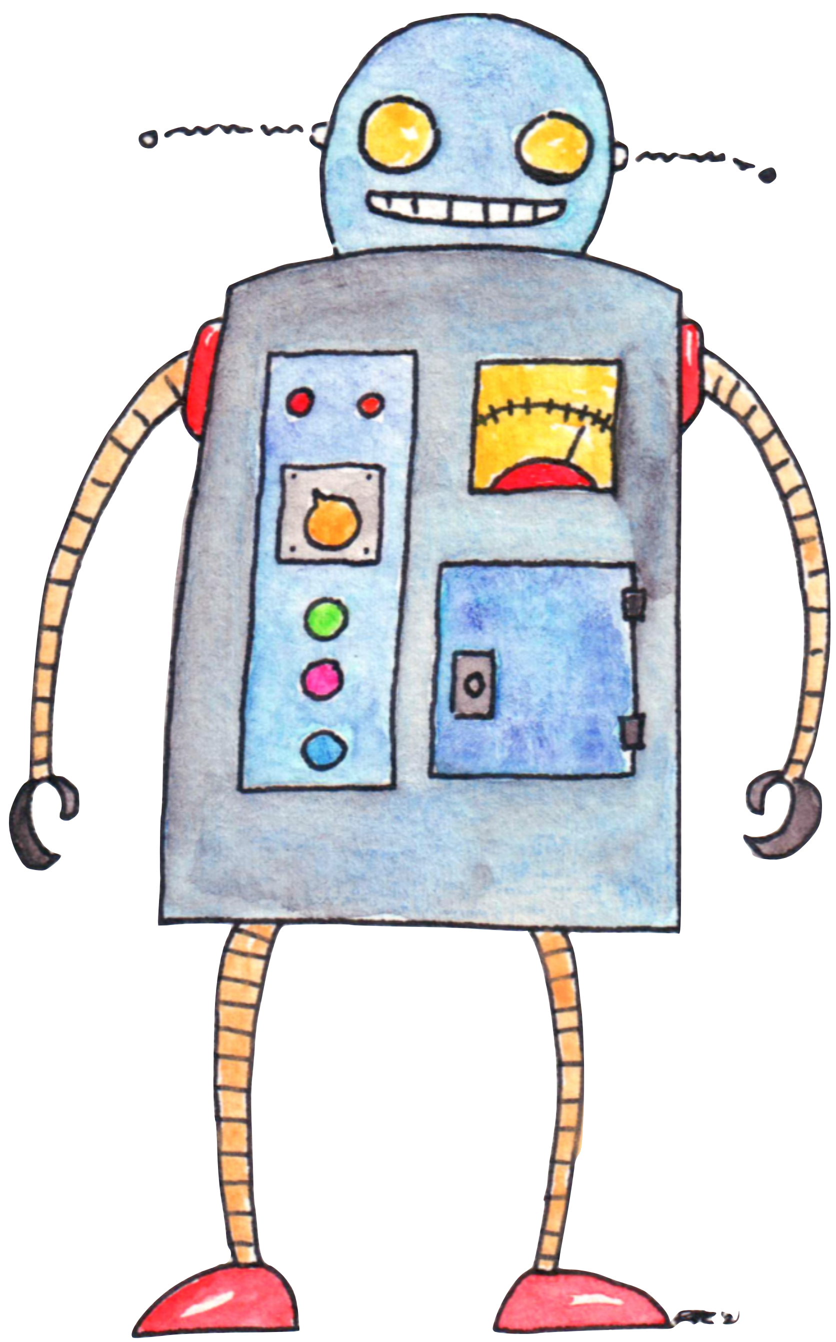
Learning Objectives

- ☐ Remember: Model-based RL framework, MLE vs VAML
- ☐ Understand: How Dyna works? Effect of model error? How to learn a model?
- ☐ Apply: Model-based RL to improve sample the efficiency of an RL agent

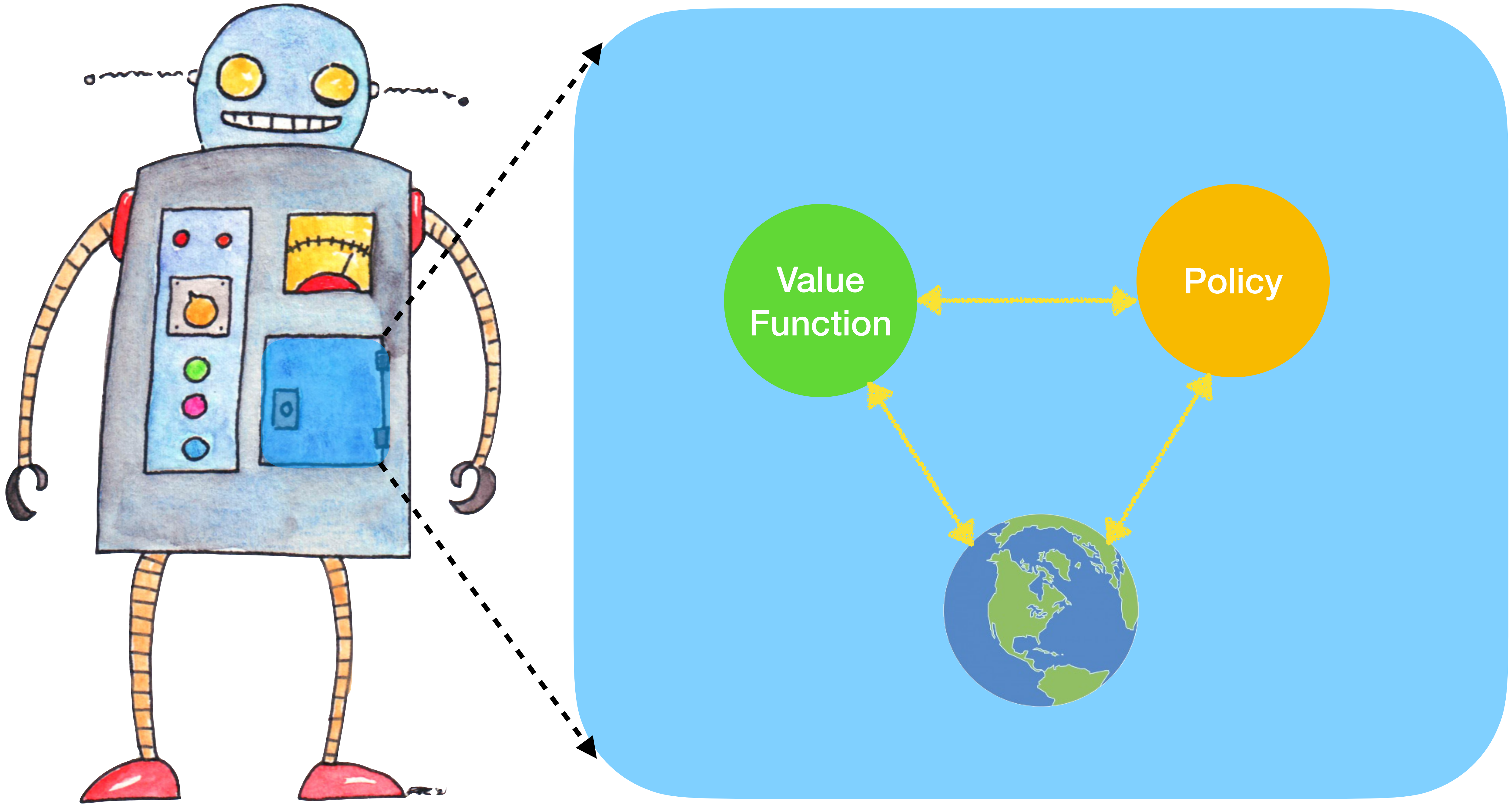
Anatomy of an RL Agent



Anatomy of an RL Agent

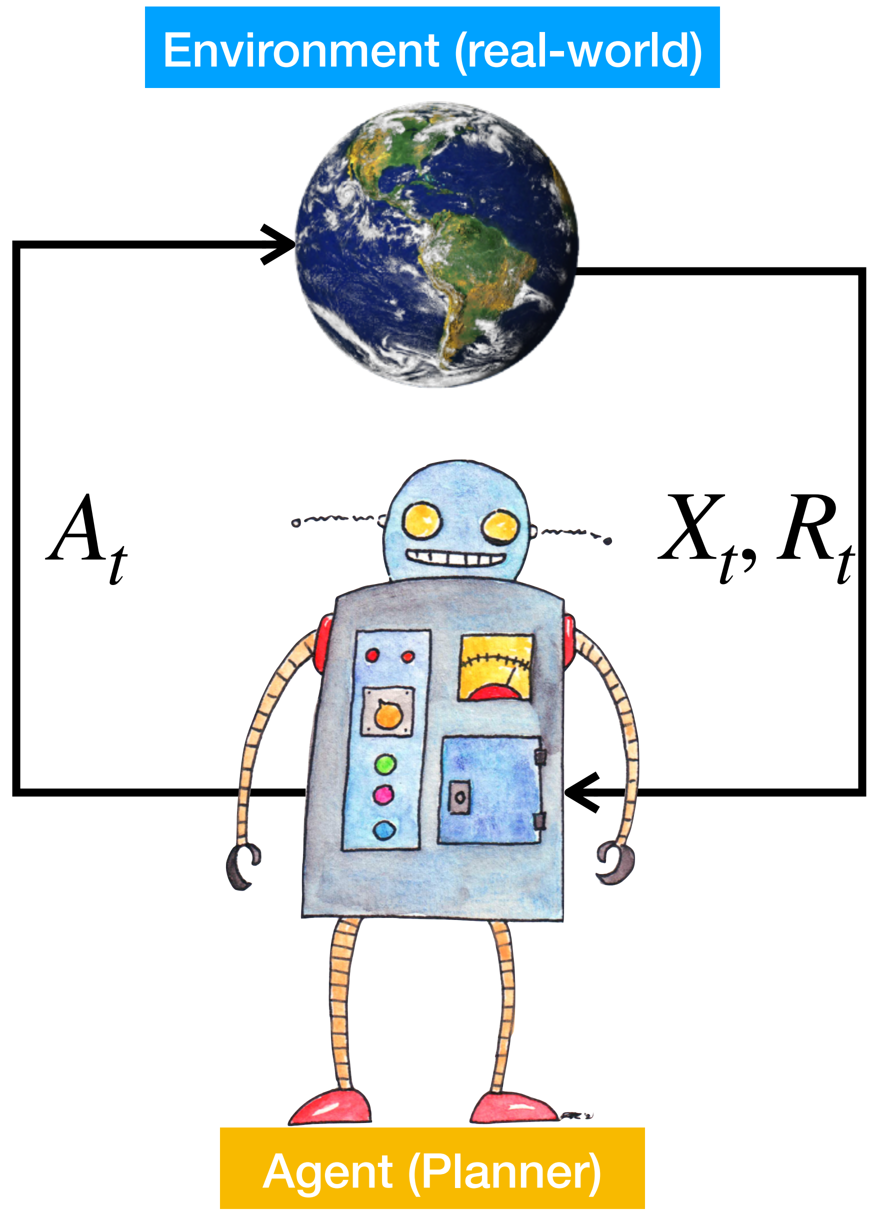


Anatomy of an RL Agent



Let us talk about model-based RL

Model-free RL Agent

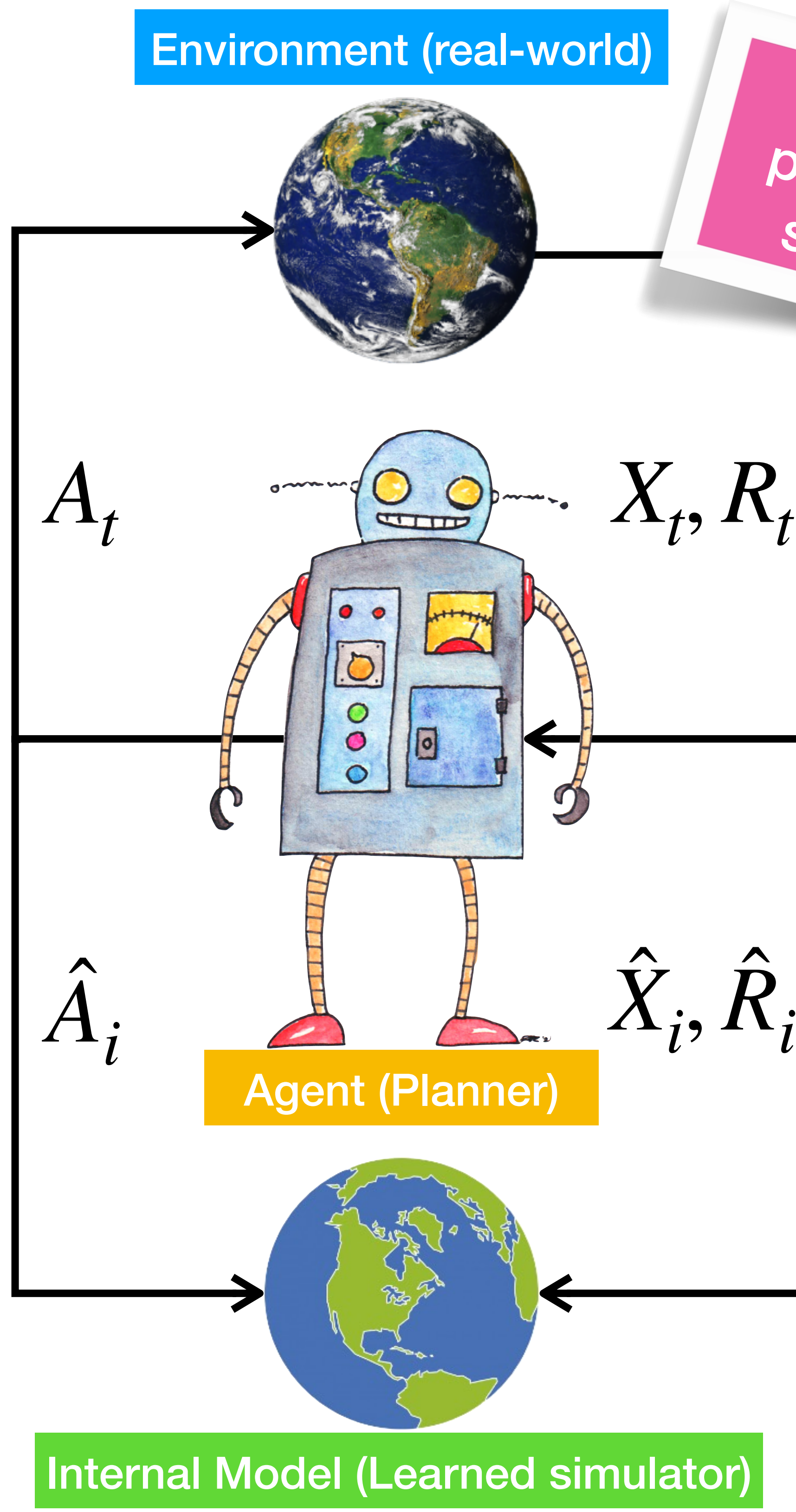


Bertsekas & Tsitsiklis, 1996 — Busoniu, Babuska, De Schutter, & Ernst, 2010 — Szepesvari, 2010 — Sutton & Barto, 2018 – Farahmand, 2021 – Meyn, 2022.

Model-based RL Agent

Two components of a MBRL agent:

- Learn a model of the environment
- Use the learned model for planning



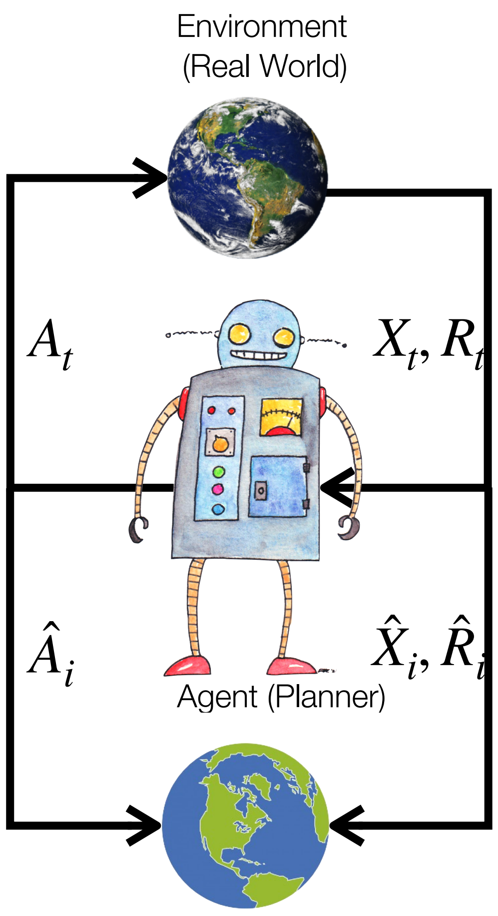
Model-based RL (MBRL) is a promising approach to design sample-efficient RL agents.

$$\tilde{X}_{i+1} \sim \hat{\mathcal{P}}(\cdot|\tilde{X}_i, \tilde{A}_t)$$
$$\tilde{R}_i \sim \hat{\mathcal{R}}(\cdot|\tilde{X}_i, \tilde{A}_i)$$

Sutton, ICML, 1990 — Peng, Williams, Adaptive Behavior, 1993 — Sutton, Szepesvári, Geramifard, and Bowling, UAI, 2008. — Parr, Li, Taylor, et al., ICML, 2008 — Deisenroth, Fox, Rasmussen, IEEE PAMI, 2015 — Levine, Finn, Darrell, Abbeel, JMLR, 2016 — Oh, Guo Lee, Lewis, Sing, NIPS, 2015 — Ha, Schmidhuber, NeurIPS, 2018.

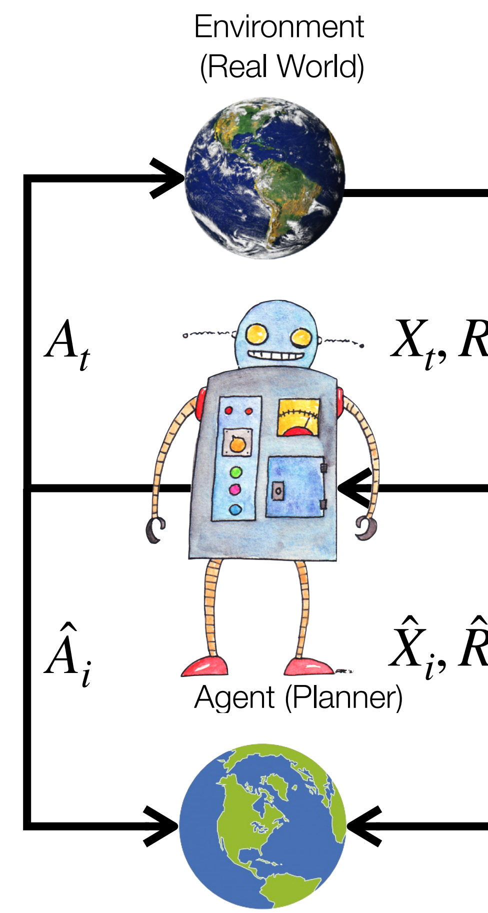
Dyna Architecture: A Prototypical MBRL Algorithm

// MDP $(\mathcal{X}, \mathcal{A}, \mathcal{R}^*, \mathcal{P}^*)$
Draw initial state $X_1 \sim \nu_{\mathcal{X}}$
for each time step t **do**
  Take action $A_t \sim \pi(\cdot|X_t)$, receive $X'_t \sim \mathcal{P}^*(\cdot|X_t, A_t)$ and $R_t \sim \mathcal{R}^*(\cdot|X_t, A_t)$.
  Update model $\hat{\mathcal{P}}$ and $\hat{\mathcal{R}}$
  Update value function and/or policy using the new sample from the real world
  for p times **do**
    Draw simulated/imaginary sample $\tilde{X}_i \sim \tilde{\nu}_{\mathcal{X}}$
    Take action $\tilde{A}_i \sim \pi(\cdot|X_t)$, receive $\tilde{X}'_i \sim \hat{\mathcal{P}}(\cdot|\tilde{X}_i, \tilde{A}_i)$
    Update value function and/or policy using the new sample from the model
  end for
  $X_{t+1} \leftarrow X'_t$
end for



Dyna Architecture: Finite State/Action Space

// MDP $(\mathcal{X}, \mathcal{A}, \mathcal{R}^*, \mathcal{P}^*, \gamma)$
// α: Learning rate for TD(0)
Draw initial state $X_1 \sim \nu_{\mathcal{X}}$
for each time step t **do**
  Take action $A_t \sim \pi(\cdot|X_t)$, receive $X'_t \sim \mathcal{P}^*(\cdot|X_t, A_t)$ and $R_t \sim \mathcal{R}^*(\cdot|X_t, A_t)$.
  MLE
  $\hat{\mathcal{P}}(x'|x, a) \leftarrow \frac{\#\{X'_i = x' | (X_i = x, A_i = a)\}}{\#\{(X_i = x, A_i = a)\}}$
  $Q(X_t, A_t) \leftarrow Q(X_t, A_t) + \alpha \left(R_t + \gamma \sum_{a \in \mathcal{A}} \pi(a|X'_t) Q(X'_t, a') - Q(X_t, A_t)\right)$
  for p times **do**
TD
    Draw simulated/imaginary sample $\tilde{X}_i \sim \tilde{\nu}_{\mathcal{X}}$
    Take action $\tilde{A}_i \sim \pi(\cdot|\tilde{X}_t)$, receive $\tilde{X}'_i \sim \hat{\mathcal{P}}(\cdot|\tilde{X}_i, \tilde{A}_i)$ and $\tilde{r}_i \leftarrow \hat{r}(\tilde{X}_i, \tilde{A}_i)$.
    $Q(\tilde{X}_i, \tilde{A}_i) \leftarrow Q(\tilde{X}_i, \tilde{A}_i) + \alpha \left(\tilde{r}_i + \gamma \sum_{a \in \pi} \pi(a|\tilde{X}'_i) Q(\tilde{X}'_i, a') - Q(\tilde{X}_i, \tilde{A}_i)\right)$
  end for
  $X_{t+1} \leftarrow X'_t$
end for



Algorithm 1 Generic Model-based Reinforcement Learning Algorithm

// MDP $(\mathcal{X}, \mathcal{A}, \mathcal{R}^*, \mathcal{P}^*, \gamma)$
// K: Number of interaction episodes
// $\mathcal{M}$: Space of transition probability kernels
// $\mathcal{G}$: Space of reward functions
Initialize a policy π_0
for $k = 0$ to $K - 1$ **do**
  Generate training set $\mathcal{D}_n^{(k)} = \{(X_i, A_i, R_i, X_i')\}_{i=1}^n$ by interacting with the true environment (potentially using π_k), i.e., $X_i' \sim \mathcal{P}^*(\cdot|X_i, A_i)$ and $R_i \sim \mathcal{R}(\cdot|X_i, A_i)$.
  $\hat{\mathcal{P}} \leftarrow \mathrm{argmin}_{\mathcal{P} \in \mathcal{M}} \mathrm{Loss}_{\mathcal{P}}(\mathcal{P}; \cup_{i=0}^k \mathcal{D}_n^{(i)})$
  $\hat{r} \leftarrow \mathrm{argmin}_{r \in \mathcal{G}} \mathrm{Loss}_{\mathcal{R}}(r; \cup_{i=0}^k \mathcal{D}_n^{(i)})$
  $\pi_{k+1} \leftarrow \mathsf{Planner}(\hat{\mathcal{P}}, \hat{\mathcal{R}})$
end for
return π_K

Why MBRL is a sensible approach?

Consider $\mathcal{P}_1$ and $\mathcal{P}_2$. The value function of policy π w.r.t. P_i is

$(I - \gamma P^\pi)V^\pi = r^\pi$

$$V^\pi_{\mathcal{P}_i} = (\mathbf{I} - \gamma \mathcal{P}^\pi_i)^{-1} r^\pi .$$

We have

$$\begin{aligned}
V^\pi_{\mathcal{P}_1} - V^\pi_{\mathcal{P}_2} &= \left[(\mathbf{I} - \gamma \mathcal{P}^\pi_1)^{-1} - (\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1}\right] (\mathbf{I} - \gamma \mathcal{P}^\pi_1) V^\pi_{\mathcal{P}_1} \\
&= \left[\mathbf{I} - (\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1} (\mathbf{I} - \gamma \mathcal{P}^\pi_1)\right] V^\pi_{\mathcal{P}_1} \\
&= \left[(\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1} (\mathbf{I} - \gamma \mathcal{P}^\pi_2) - (\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1} (\mathbf{I} - \gamma \mathcal{P}^\pi_1)\right] V^\pi_{\mathcal{P}_1} \\
&= (\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1} \left[(\mathbf{I} - \gamma \mathcal{P}^\pi_2) - (\mathbf{I} - \gamma \mathcal{P}^\pi_1)\right] V^\pi_{\mathcal{P}_1} \\
&= \gamma (\mathbf{I} - \gamma \mathcal{P}^\pi_2)^{-1} (\mathcal{P}^\pi_1 - \mathcal{P}^\pi_2) V^\pi_{\mathcal{P}_1}
\end{aligned}$$

$\|(I - A)^{-1}\|_\infty \le \frac{1}{1 - \|A\|_\infty}$

If $\|A\|_\infty < 1$

$\|A \cdot B\|_\infty \le \|A\|_\infty \cdot \|B\|_\infty$

$\|A \cdot u\|_\infty \le \|A\|_\infty \cdot \|u\|_\infty$

Therefore, if we have an approximate model $\hat{\mathcal{P}} \approx \mathcal{P}$, for any policy π, we get that

$$
\begin{aligned}
V^\pi_{\mathcal{P}} - V^\pi_{\hat{\mathcal{P}}} &= \gamma(\mathbf{I} - \gamma\hat{\mathcal{P}}^\pi)^{-1}(\mathcal{P}^\pi - \hat{\mathcal{P}}^\pi)V^\pi_{\mathcal{P}}, \\
\Rightarrow \left\| V^\pi_{\mathcal{P}} - V^\pi_{\hat{\mathcal{P}}} \right\|_\infty &\le \gamma \left\| (\mathbf{I} - \gamma\hat{\mathcal{P}}^\pi)^{-1}(\mathcal{P}^\pi - \hat{\mathcal{P}}^\pi)V^\pi_{\mathcal{P}} \right\|_\infty \\
&\le \gamma \left\| \boxed{(\mathbf{I} - \gamma\hat{\mathcal{P}}^\pi)^{-1}} \right\|_\infty \left\| (\mathcal{P}^\pi - \hat{\mathcal{P}}^\pi)V^\pi_{\mathcal{P}} \right\|_\infty \\
&\le \frac{\gamma}{1 - \gamma \left\| \hat{\mathcal{P}}^\pi \right\|_\infty} \left\| (\mathcal{P}^\pi - \hat{\mathcal{P}}^\pi)V^\pi_{\mathcal{P}} \right\|_\infty \\
&\le \frac{\gamma}{1 - \gamma} \boxed{\left\| \mathcal{P}^\pi - \hat{\mathcal{P}}^\pi \right\|_\infty \|V^\pi_{\mathcal{P}}\|_\infty} \\
&\le \frac{\gamma R_{\max}}{(1 - \gamma)^2} \left\| \mathcal{P}^\pi - \hat{\mathcal{P}}^\pi \right\|_\infty .
\end{aligned}
$$

$\|V^\pi\|_\infty \le \frac{R_{max}}{1-\gamma}$

Suppose that we compute the optimal policy within $\hat{\mathcal{P}}$. Let's call it $\hat{\pi}^*$. Also consider the optimal policy π^* for $\mathcal{P}$. We now execute policy $\hat{\pi}^*$ in $\mathcal{P}$. In general, the policy $\hat{\pi}^*$ is not an optimal policy for $\mathcal{P}$. How worse can it be compared to the actual optimal policy π^*? We have

$$
\begin{aligned}
0 \leq V_{\mathcal{P}}^{\pi^*} - V_{\mathcal{P}}^{\hat{\pi}^*} &= V_{\mathcal{P}}^{\pi^*} - V_{\hat{\mathcal{P}}}^{\pi^*} + \underbrace{V_{\hat{\mathcal{P}}}^{\pi^*} - V_{\hat{\mathcal{P}}}^{\hat{\pi}^*}}_{\leq 0} + V_{\hat{\mathcal{P}}}^{\hat{\pi}^*} - V_{\mathcal{P}}^{\hat{\pi}^*} \\
&\leq \left(V_{\mathcal{P}}^{\pi^*} - V_{\hat{\mathcal{P}}}^{\pi^*}\right) + \left(V_{\hat{\mathcal{P}}}^{\hat{\pi}^*} - V_{\mathcal{P}}^{\hat{\pi}^*}\right).
\end{aligned}
$$

By the previous result, we have

$$
\begin{aligned}
V_{\mathcal{P}}^{\pi^*} - V_{\hat{\mathcal{P}}}^{\pi^*} &= \gamma(\mathbf{I} - \gamma\hat{\mathcal{P}}^{\pi^*})^{-1}(\mathcal{P}^{\pi^*} - \hat{\mathcal{P}}^{\pi^*})V_{\mathcal{P}}^{\pi^*} \\
V_{\hat{\mathcal{P}}}^{\hat{\pi}^*} - V_{\mathcal{P}}^{\hat{\pi}^*} &= \gamma(\mathbf{I} - \gamma\hat{\mathcal{P}}^{\hat{\pi}^*})^{-1}(\hat{\mathcal{P}}^{\hat{\pi}^*} - \mathcal{P}^{\hat{\pi}^*})V_{\mathcal{P}}^{\hat{\pi}^*}
\end{aligned}
$$

Therefore,

$$
0 \leq V_{\mathcal{P}}^{\pi^*} - V_{\mathcal{P}}^{\hat{\pi}^*} \leq \gamma\left[(\mathbf{I} - \gamma\hat{\mathcal{P}}^{\pi^*})^{-1}\underbrace{(\mathcal{P}^{\pi^*} - \hat{\mathcal{P}}^{\pi^*})}_{1}V_{\mathcal{P}}^{\pi^*} + (\mathbf{I} - \gamma\hat{\mathcal{P}}^{\hat{\pi}^*})^{-1}\underbrace{(\hat{\mathcal{P}}^{\hat{\pi}^*} - \mathcal{P}^{\hat{\pi}^*})}_{2}V_{\mathcal{P}}^{\hat{\pi}^*}\right].
$$

Simulation Lemma

$$0 \le V_{\mathcal{P}}^{\pi^*} - V_{\mathcal{P}}^{\hat{\pi}^*} \le \gamma \left[(\mathbf{I} - \gamma \hat{\mathcal{P}}^{\pi^*})^{-1} (\mathcal{P}^{\pi^*} - \hat{\mathcal{P}}^{\pi^*}) V_{\mathcal{P}}^{\pi^*} + (\mathbf{I} - \gamma \hat{\mathcal{P}}^{\hat{\pi}^*})^{-1} (\hat{\mathcal{P}}^{\hat{\pi}^*} - \mathcal{P}^{\hat{\pi}^*}) V_{\mathcal{P}}^{\hat{\pi}^*} \right].$$

Take the supremum norm:

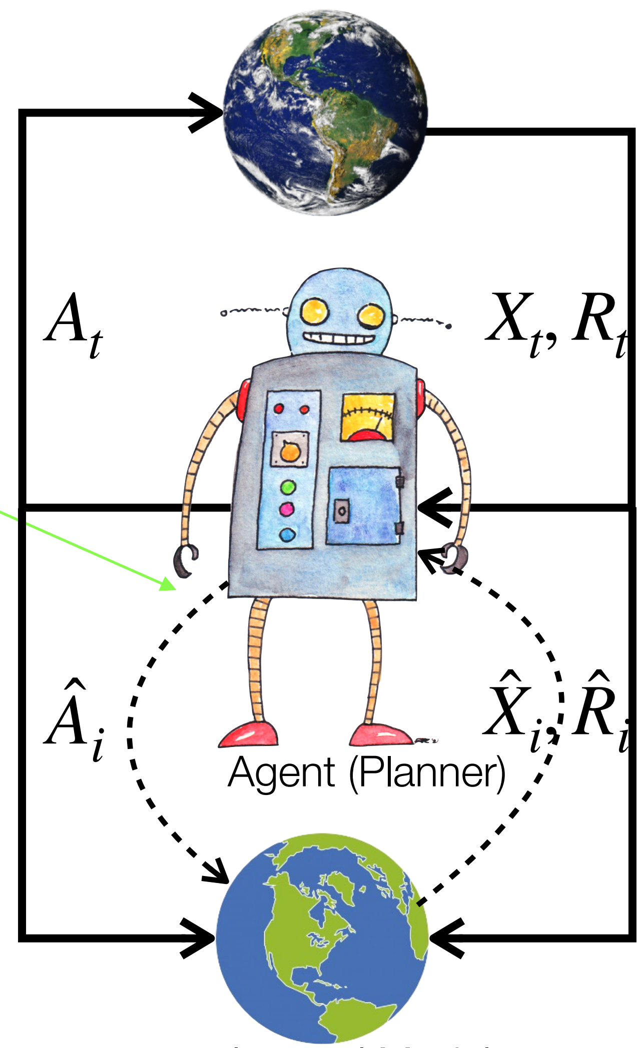
$$\begin{aligned}
\left\| V_{\mathcal{P}}^{\pi^*} - V_{\mathcal{P}}^{\hat{\pi}^*} \right\|_\infty &\le \frac{\gamma}{1-\gamma} \left[\left\| (\mathcal{P}^{\pi^*} - \hat{\mathcal{P}}^{\pi^*}) V_{\mathcal{P}}^{\pi^*} \right\|_\infty + \left\| (\hat{\mathcal{P}}^{\hat{\pi}^*} - \mathcal{P}^{\hat{\pi}^*}) V_{\mathcal{P}}^{\hat{\pi}^*} \right\|_\infty \right] \\
&\le \frac{2\gamma V_{\max}}{1-\gamma} \max_{\pi \in \{\pi^*, \hat{\pi}^*\}} \left\| \mathcal{P}^\pi - \hat{\mathcal{P}}^\pi \right\|_\infty \\
&\le \frac{2\gamma R_{\max}}{(1-\gamma)^2} \left\| \mathcal{P} - \hat{\mathcal{P}} \right\|_\infty .
\end{aligned}$$

Issues in MBRL

Choice of Planner

Planner

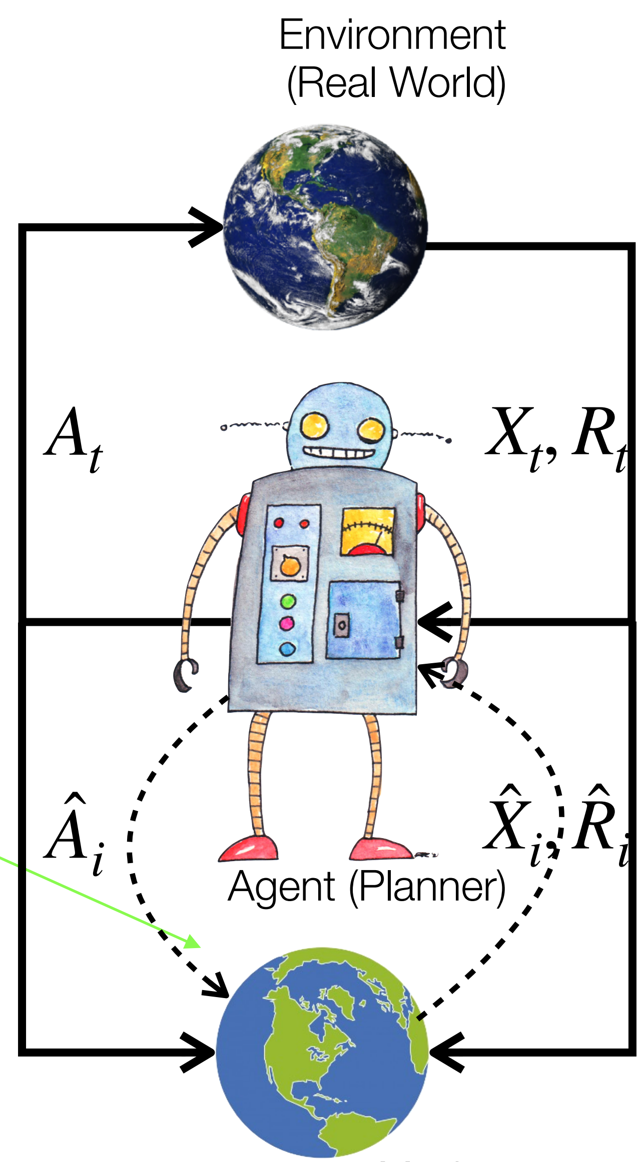
- Value-based
- Policy Search



Policy Search

- PILCO: Marc P. Deisenroth, Dieter Fox, and Carl E. Rasmussen, "Gaussian processes for data-efficient learning in robotics and control," IEEE Trans. on PAMI, 2015.
- GPS: - Sergey Levine and Pieter Abbeel, "Learning neural network policies with guided policy search under unknown dynamics," NIPS, 2014.

Model Learning



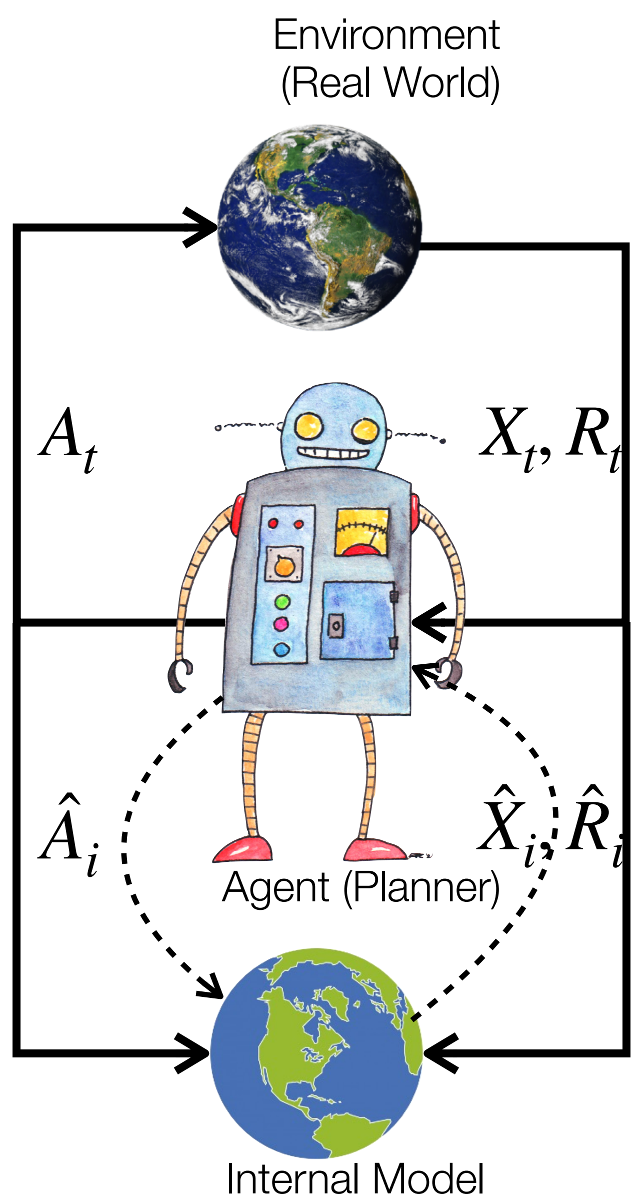
Model Learning

- MLE
- Bayesian
- Decision-Aware Model Learning

Decision-Aware Model Learning

* AMF, André M.S. Barreto, and Daniel N. Nikovski, "Value-aware model learning for reinforcement learning," AISTATS, 2017.
* David Silver, Hado van Hasselt, Matteo Hessel, et al., "The Predictron: End-to-end learning and planning," ICML, 2017.
* Junhyuk Oh, Satinder Singh, and Honglak Lee, "Value prediction network," NIPS, 2017.
* Joshua Joseph, Alborz Geramifard, John W Roberts, Jonathan P How, and Nicholas Roy, "Reinforcement learning with misspecified model classes," ICRA, 2013.

Distribution Mismatch



$\rho\mathcal{P}^*$ $\rho\mathcal{P}^{*2}$ $\rho\mathcal{P}^{*3}$

$\rho\hat{\mathcal{P}}$ $\rho\hat{\mathcal{P}}^2$ $\rho\hat{\mathcal{P}}^3$

Distribution error

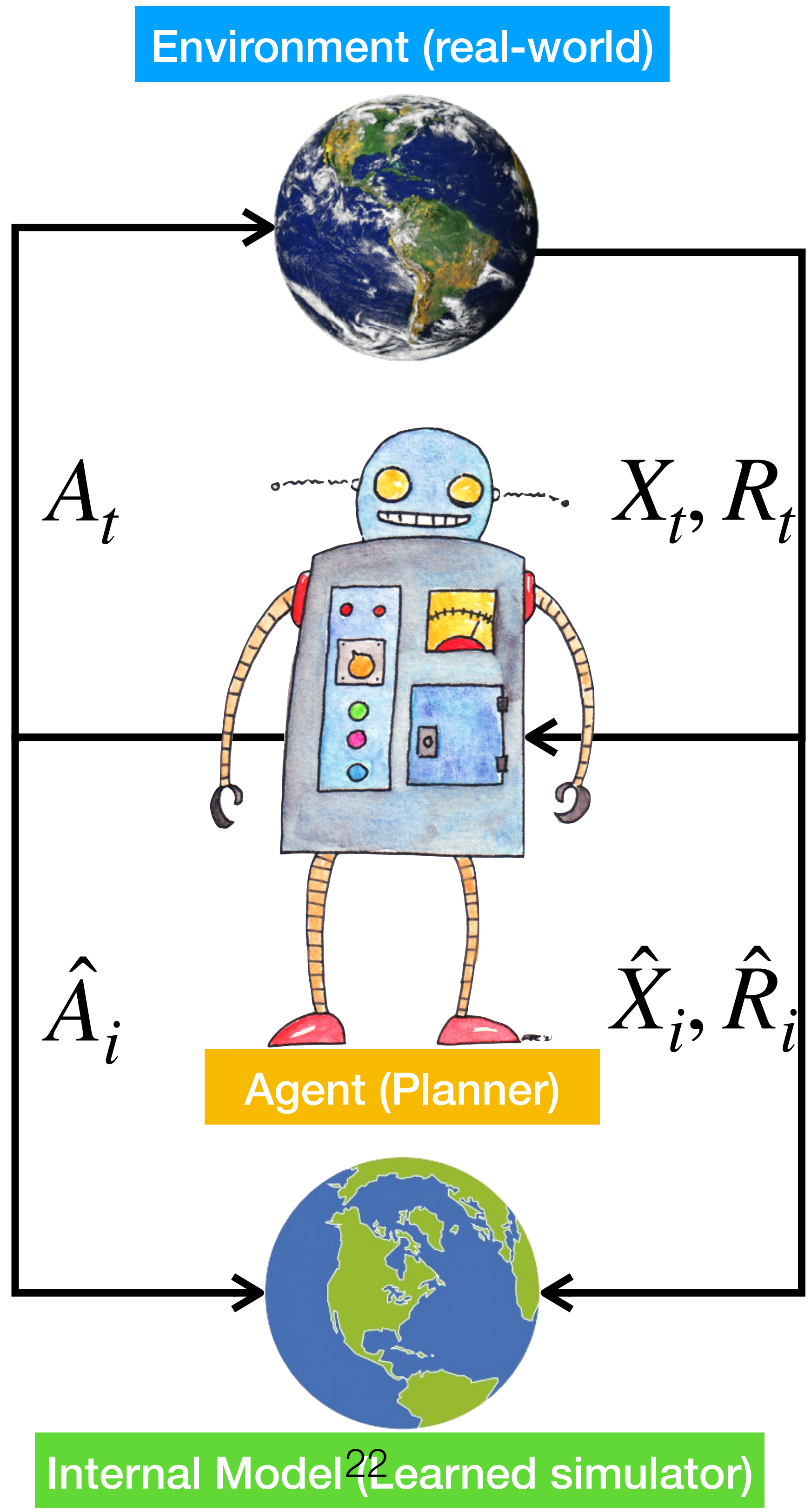
Distribution Mismatch in MBRL

- Erin Talvitie, “Self-correcting models for model-based reinforcement learning,” AAAI, 2017.
- Erik Talvitie, “Model regularization for stable sample rollouts,” UAI, 2014.
- Arun Venkatraman, Martial Hebert, and J. Andrew Bagnell, “Improving multi-step prediction of learned time series models,” AAAI, 2015.

Model-based RL Agent

Two components of a MBRL agent:

- Learn a model of the environment
- Use the learned model for planning



$$\tilde{X}_{i+1} \sim \hat{\mathcal{P}}(\cdot|\tilde{X}_i, \tilde{A}_t)$$
$$\tilde{R}_i \sim \hat{\mathcal{R}}(\cdot|\tilde{X}_i, \tilde{A}_i)$$

How should we learn a good model for model-based RL?

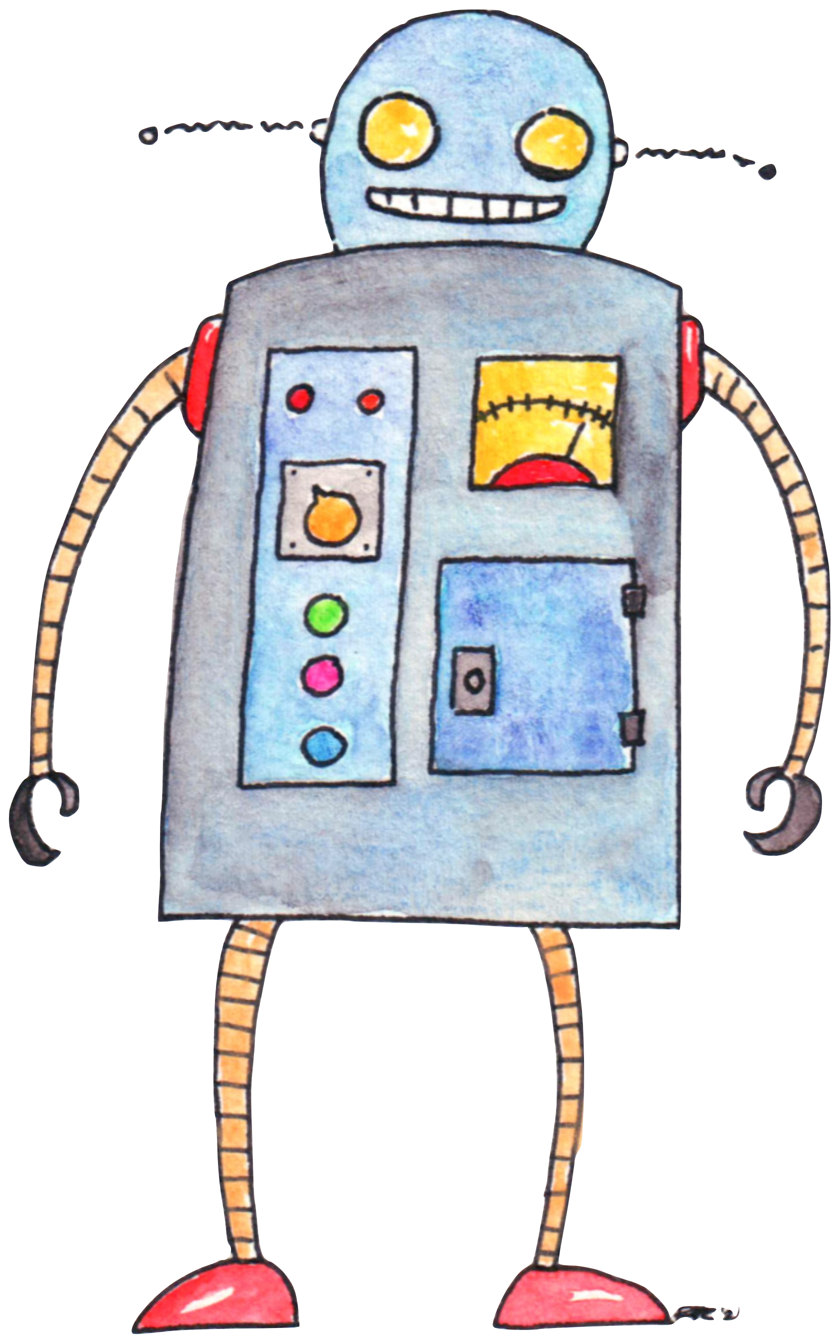
The conventional approach to model learning might be an overkill!

Conventional Approaches to Model Learning

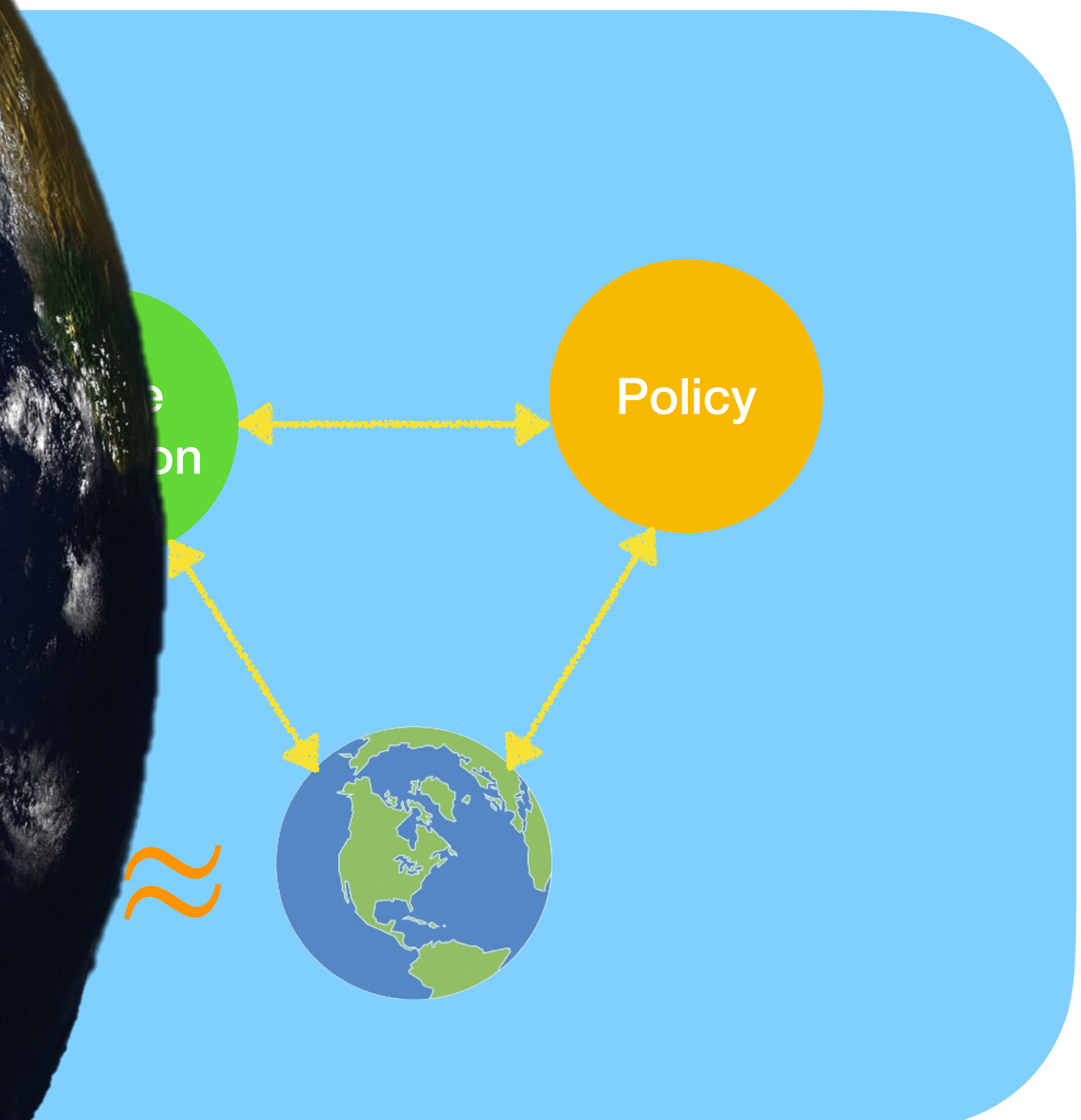
Learn a predictive model that captures **all aspects** of the environment as much as possible.

- Maximum Likelihood Estimate (MLE)
- Bayesian Inference

An RL Agent in a Big World



t in a Big World



What should a little agent do in a big world?

An RL Agent in a Big World: Adaptive Agents (Adage) Lab's Approach

- Decision-Aware Model Learning: Learning models that matter for decision making
  - Value-Aware Model Learning (VAML), Policy-Aware Model Learning (PAML), Distributional Equivalent, and several practical variants.
    - Much efforts by others in recent years: Predictron, VPN , TreeQN, GAMPS, OMD, Value-targeted regression, muZero, Value Equivalent (Sampling), etc.
- Continual Knowledge Re-Grounding (KnoReG): Benefit from an approximate model as much as possible, and continually correct the knowledge by re-grounding to the real world.
- Others: Continual Learning and Meta-Learning (ex. Alberta AI Plan)

Not all aspects are equally needed!

The world is often needlessly too complex anyway (Big World hypothesis)

Artist Robot

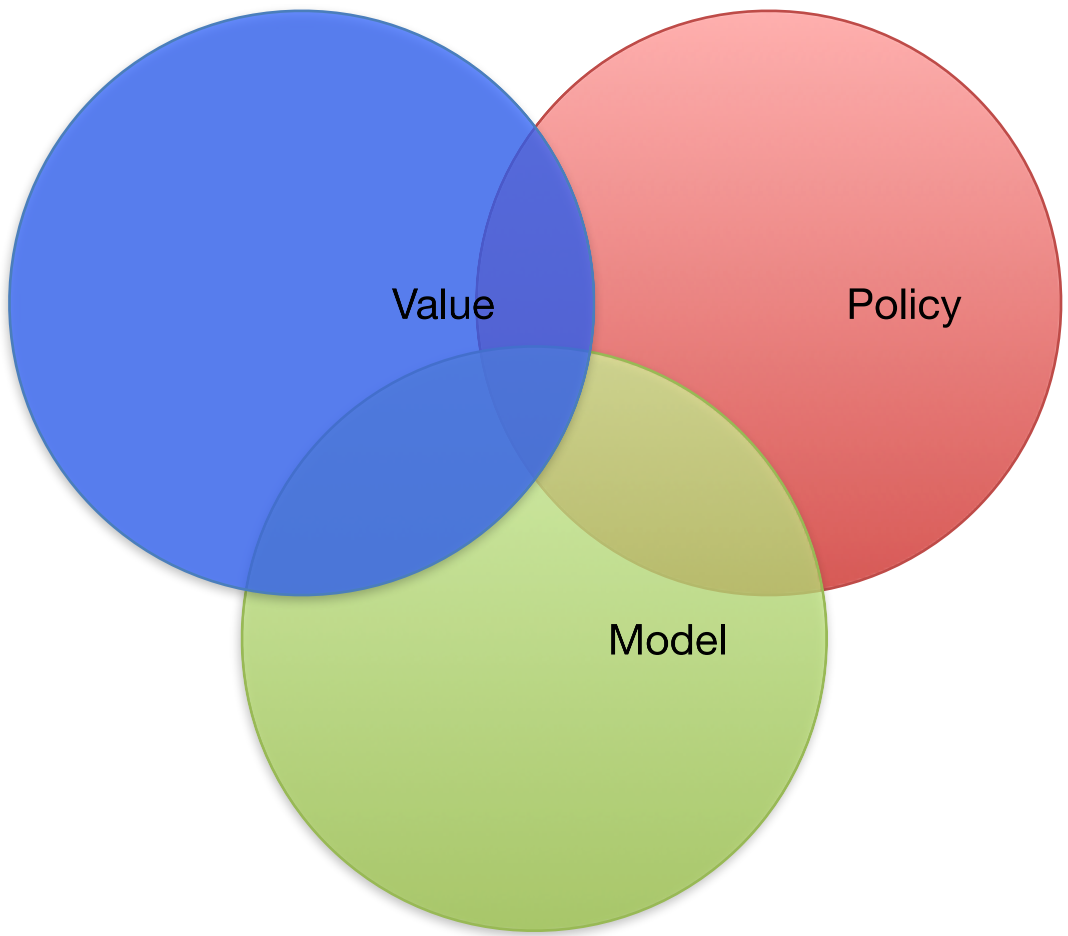
Cleaning Robot

The world might be the same, but the tasks are not!

The conventional approach to model learning might be an overkill!

erft); Cleaning Robot (Mark H. Evans)

How to incorporate information about the decision problem/ task into the model learning process itself?



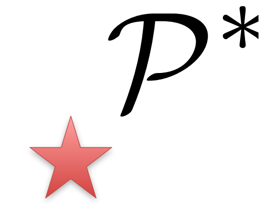
We have to pay attention to the interaction of model and the value function or policy.

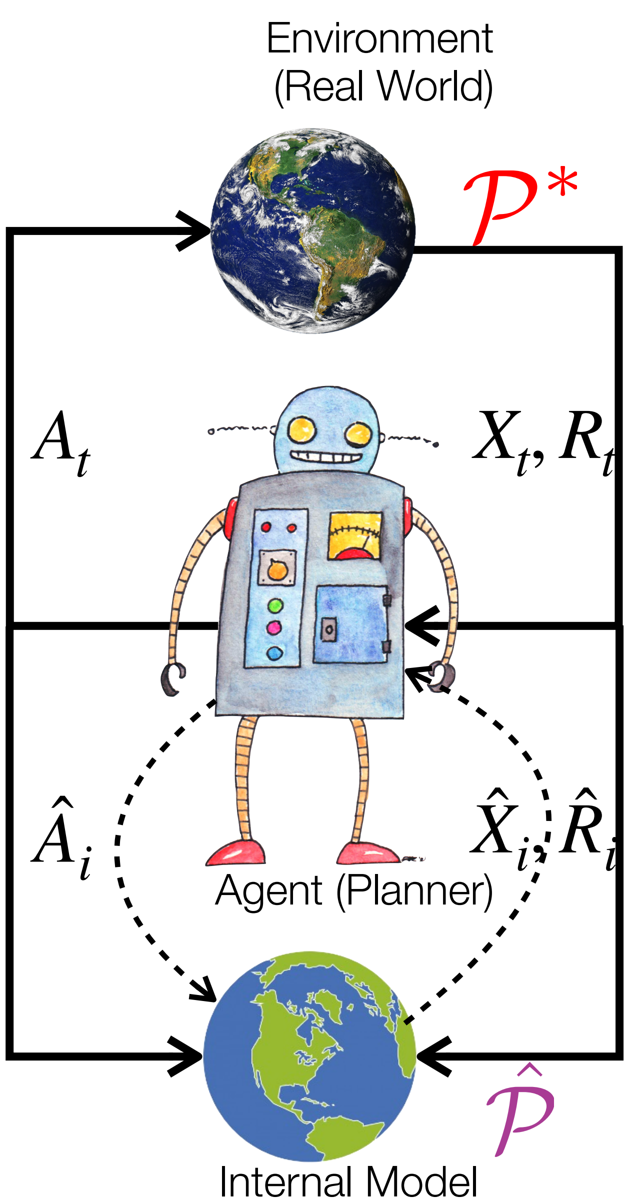
Decision-Aware Model Learning

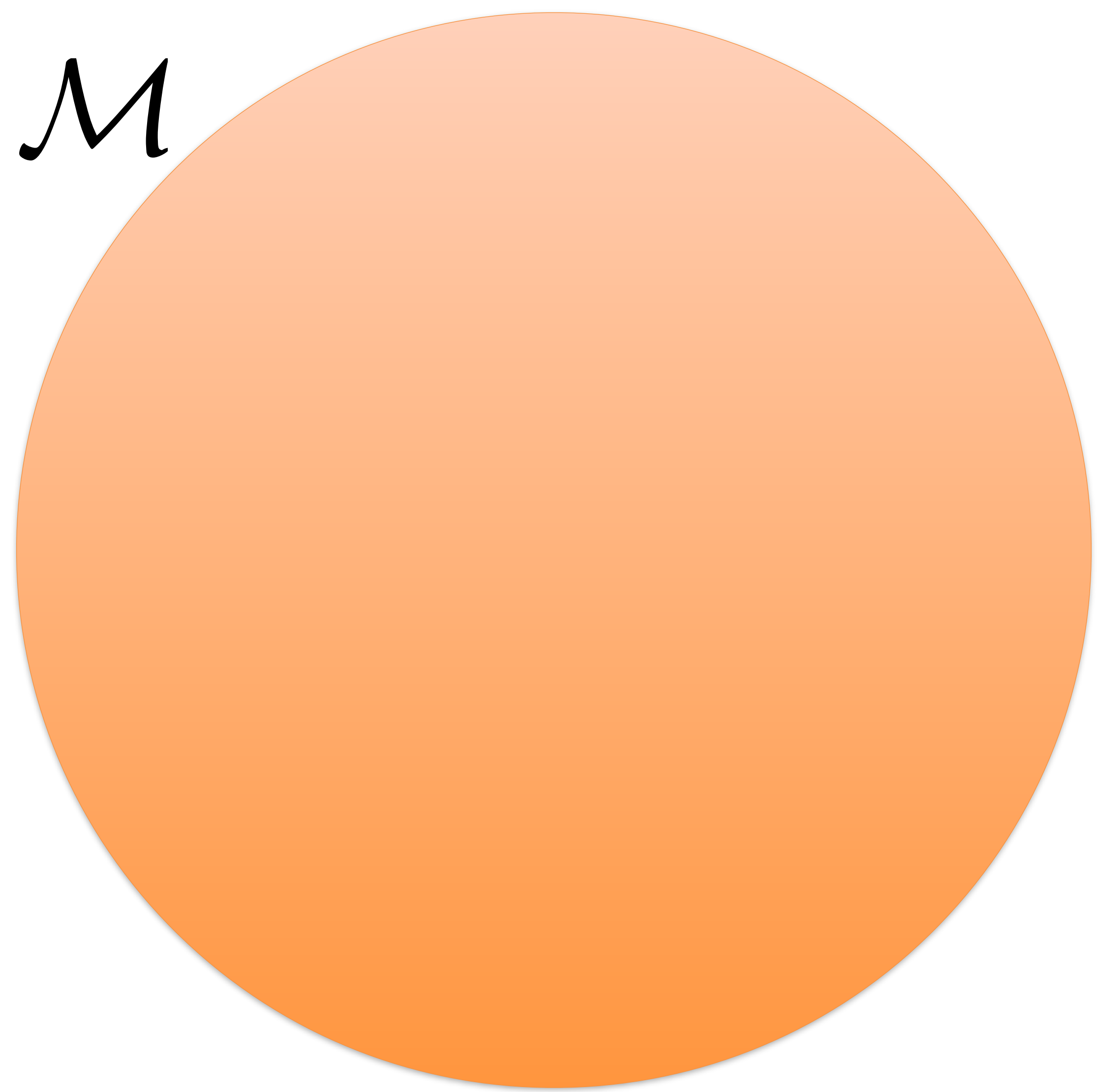
AMF, Barreto, Nikovski, “Value-Aware Loss Function for Model Learning in Reinforcement Learning,” European Workshop on Reinforcement Learning (EWRL), 2016.
AMF, Barreto, Nikovski, “Value-Aware Loss Function for Model-Based Reinforcement Learning,” Artificial Intelligence and Statistics (AISTATS), 2017.
AMF, “Iterative Value-Aware Model Learning,” Neural Information Processing Systems (NeurIPS), 2018.
Abachi, Ghavamzadeh, **AMF**, “Policy-Aware Model Learning for Policy Gradient Methods,” preprint, 2020.
Voelcker, Liao, Garg, **AMF**, “Value Gradient Weighted Model-Based Reinforcement Learning,” International Conference on Learning Representation (ICLR), 2022.
Kastner, Erdogdu, **AMF**, “Distributional Model Equivalence for Risk-Sensitive Reinforcement Learning,” Neural Information Processing Systems (NeurIPS), 2023.
Voelcker, Pedan, Ahmadian, Abachi, Gilitschenski, **AMF**, “Calibrated Value-Aware Model Learning with Probabilistic Environment Models,” International Conference on Machine Learning (ICML), 2025.

Let us try to design a **decision-aware** model learning method!

- True model of the environment: $\mathcal{P}^*$
- We are given a dataset $\mathcal{D}_n = \{(X_i, A_i, X_i')\}_{i=1}^n$ with $Z_i = (X_i, A_i) \sim \nu(\mathcal{X} \times \mathcal{A})$ and $X_i' \sim \mathcal{P}^*(\cdot|X_i, A_i)$
- Policy of the MBRL: $\pi \leftarrow \mathsf{Planner}(\hat{\mathcal{P}})$
- How to estimate a model of the environment $\hat{\mathcal{P}}$ such that π is a high-performing policy?







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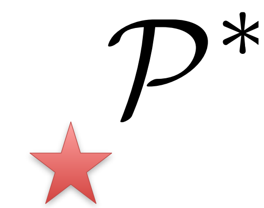

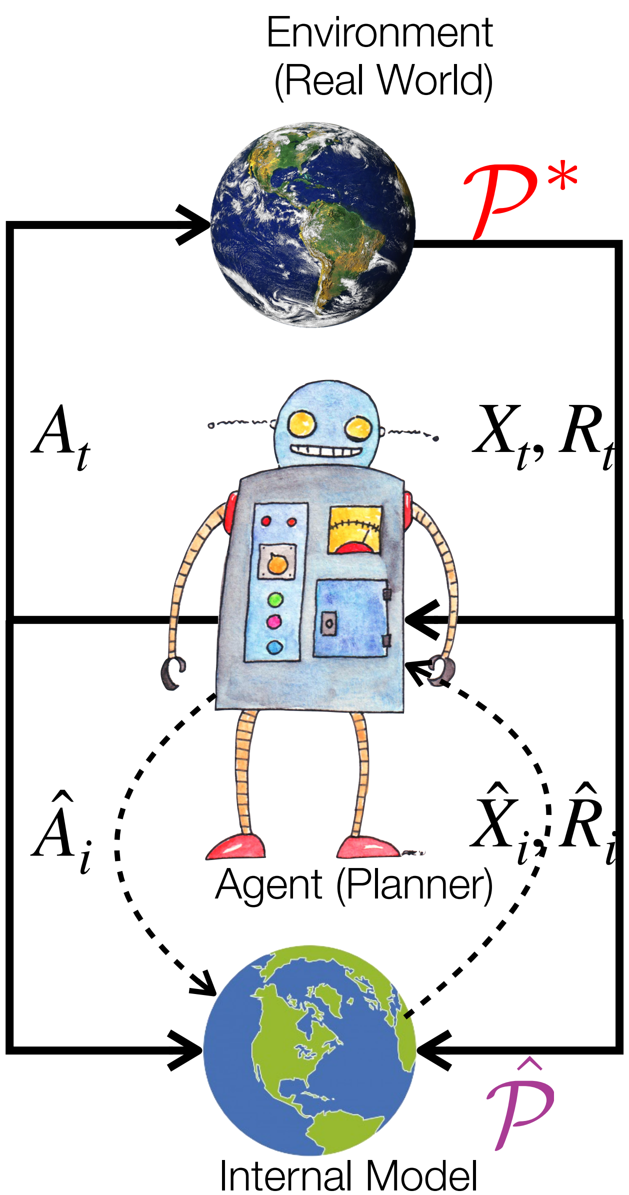

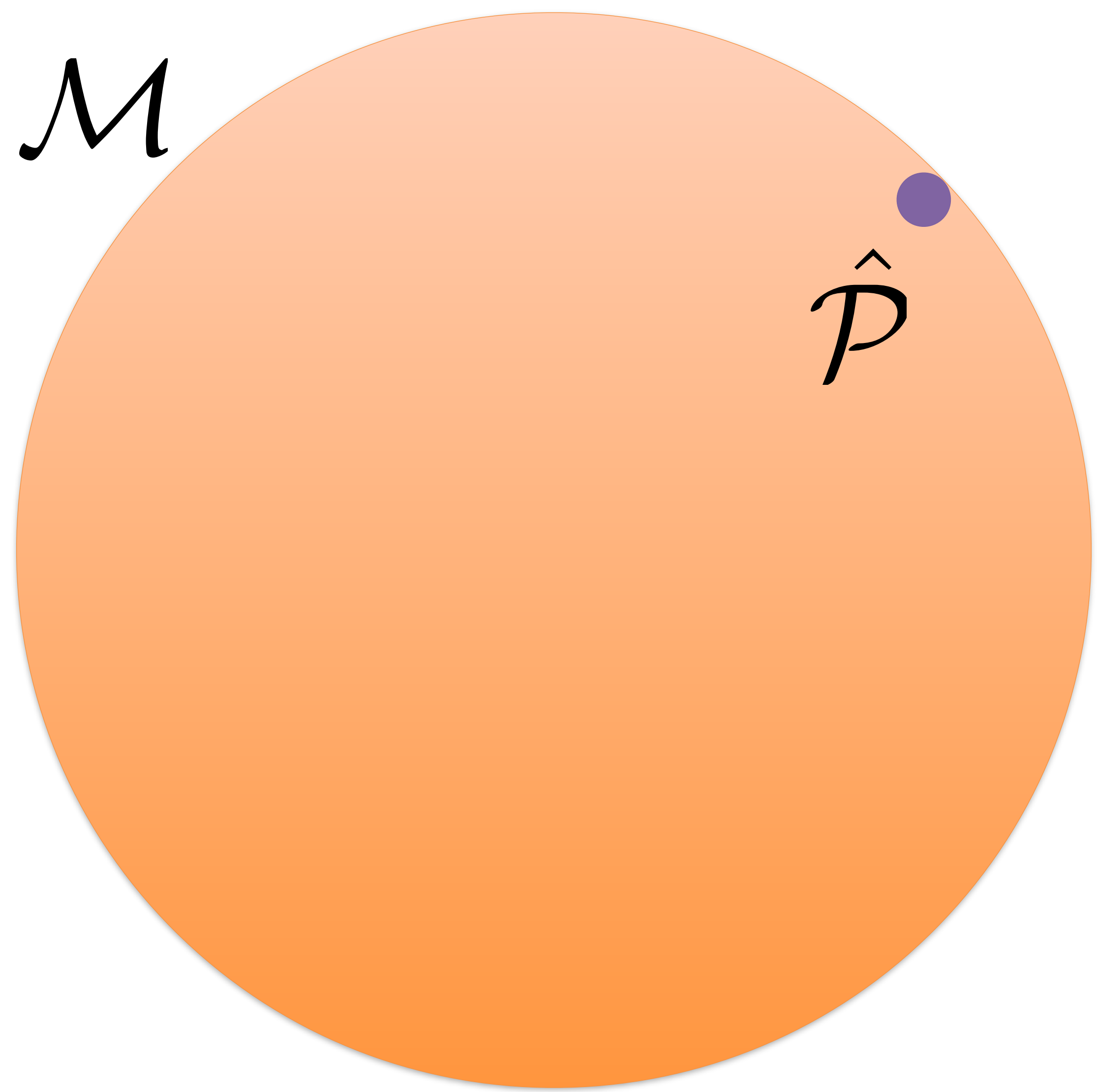

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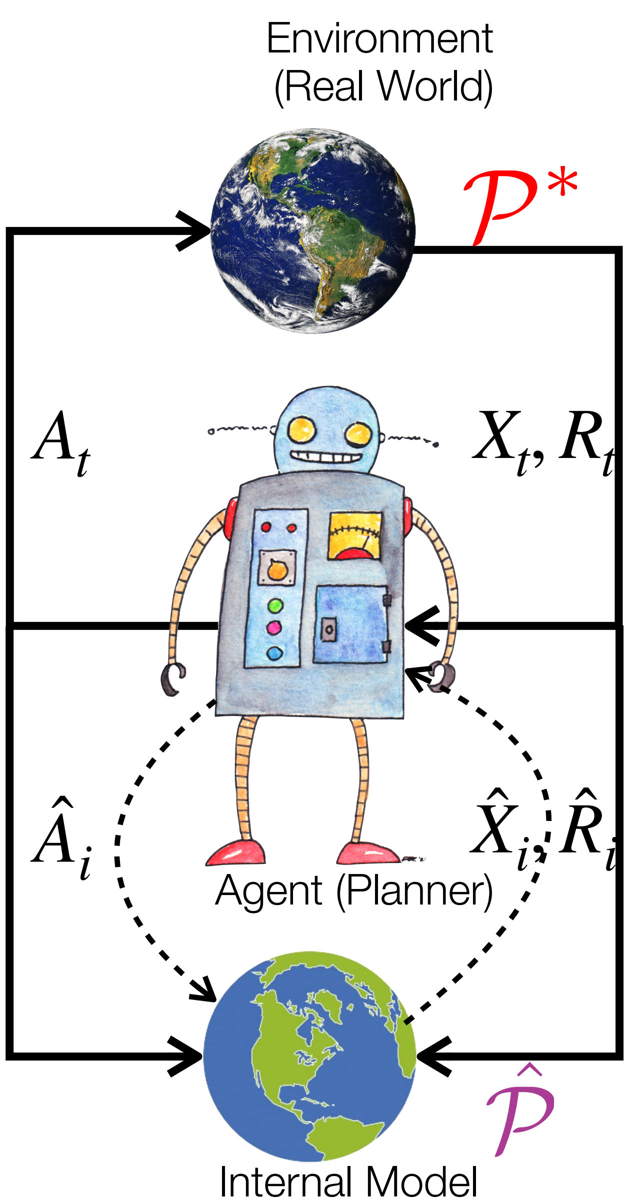

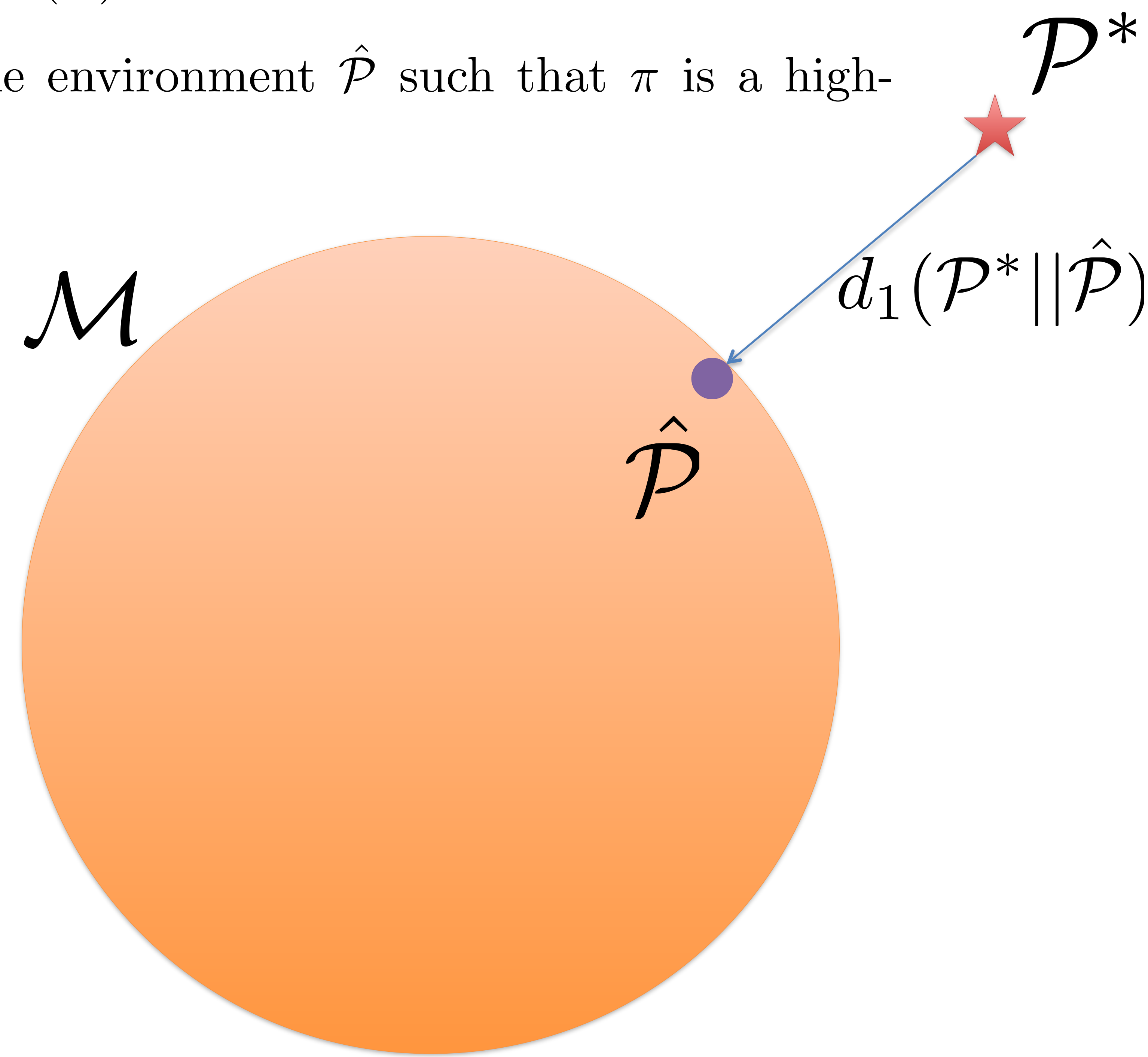

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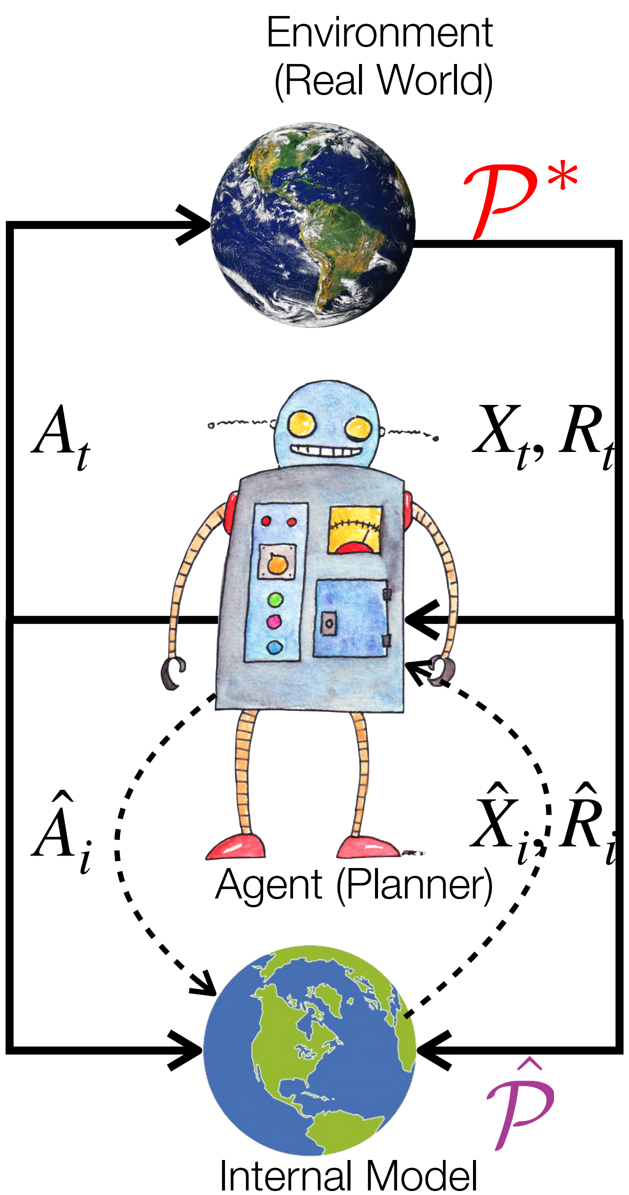
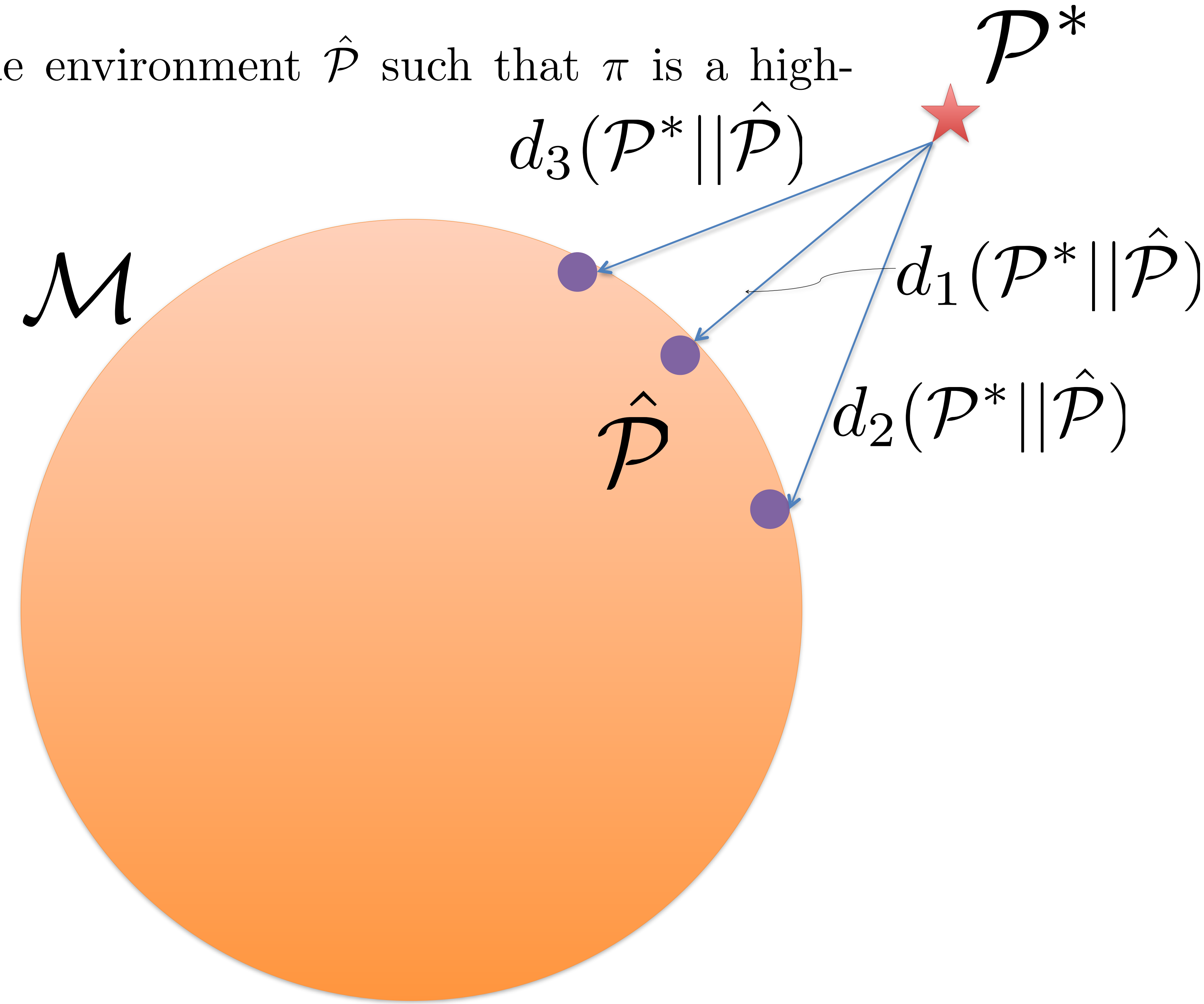
- Policy of the MBRL: $\pi \leftarrow \mathsf{Planner}(\hat{\mathcal{P}})$
- How to estimate a model of the environment $\hat{\mathcal{P}}$ such that π is a high-performing policy?

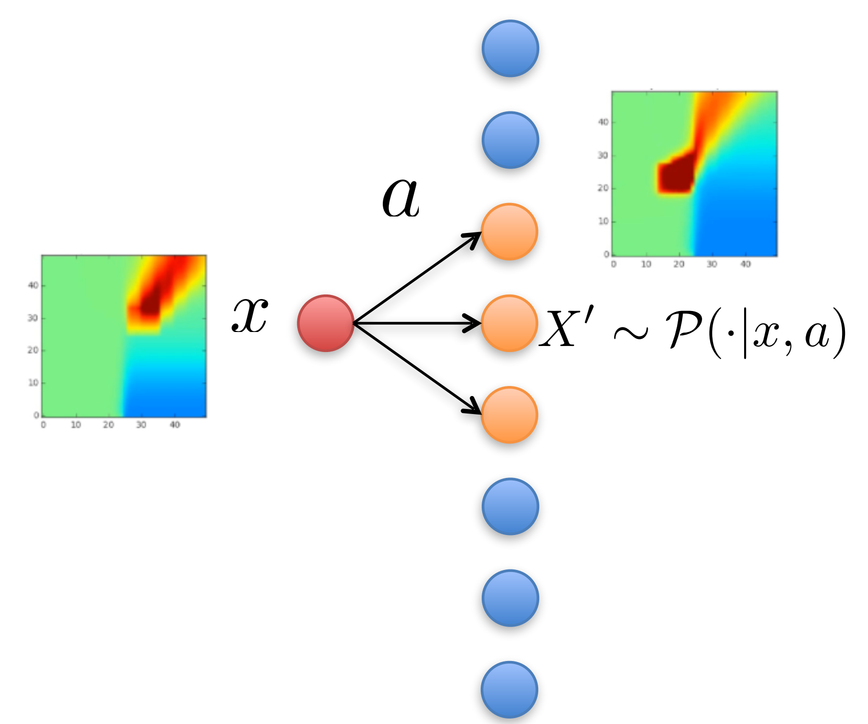




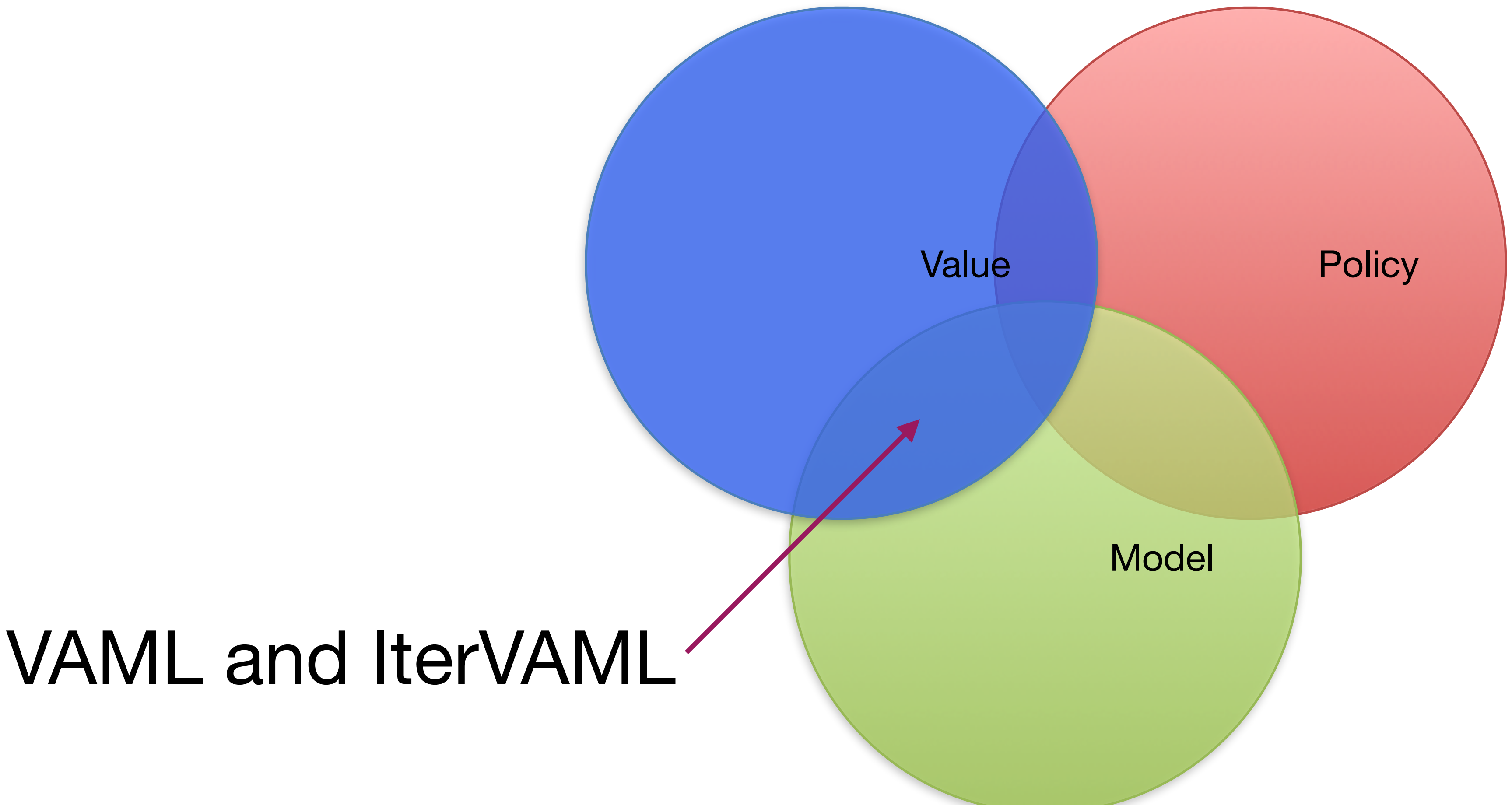
What kind of Planner?

- Variety of Planners
  - Value-based, Policy Gradient (vanilla, Natural, TRPO, etc).
- Let's focus on Bellman operator-based ones (ex. Value Iteration, TD, DQN):

$$T^*_{\mathcal{P}} Q(x,a) = r(x,a) + \gamma \int \mathcal{P}(\mathrm{d}x'|x,a) \max_{a' \in \mathcal{A}} Q(x',a')$$



Value-Aware Model Learning (VAML)



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Voelcker, Pedan, Ahmadian, Abachi, Gilitschenski, **AMF**, "Calibrated Value-Aware Model Learning with Probabilistic Environment Models," International Conference on Machine Learning (ICML), 2025.

Value-Aware Model Learning

Goal:

Finding a model that "preserves" the effect of the **Bellman operator** as much as possible.

$$T^*_{\hat{\mathcal{P}}} Q \approx T^*_{\mathcal{P}^*} Q$$

Bellman operator w.r.t. the learned model

Bellman operator w.r.t. the true model

$$T^*_{\mathcal{P}} Q(x, a) = r(x, a) + \gamma \int \mathcal{P}(\mathrm{d}x' | x, a) \max_{a' \in \mathcal{A}} Q(x', a')$$

Value-Aware Model Learning

Let us construct a new loss function …

$$T^*_{\hat{\mathcal{P}}} Q \approx T^*_{\mathcal{P}^*} Q$$

Value-Aware Model Learning

$$T^*_{\mathcal{P}^*} Q(x,a) = r(x,a) + \gamma \int \mathcal{P}^*(\mathrm{d}x'|x,a) \max_{a' \in \mathcal{A}} Q(x',a')$$

$$T^*_{\hat{\mathcal{P}}} Q(x,a) = r(x,a) + \gamma \int \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \max_{a' \in \mathcal{A}} Q(x',a')$$

$$\Big\Downarrow \; T^*_{\hat{\mathcal{P}}} Q \approx T^*_{\mathcal{P}^*} Q$$

$$\begin{aligned} c(\hat{\mathcal{P}}, \mathcal{P}^*; V)(x,a) &= \left| \left\langle \mathcal{P}^*(\cdot|x,a) - \hat{\mathcal{P}}(\cdot|x,a) \,,\, V \right\rangle \right| \\ &= \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right| \end{aligned}$$

Maximum Likelihood Estimator

Let P_1, P_2 be defined over $\mathcal{X}$ (just to simplify). Note that

$$\|P_1 - P_2\|_1 \le \sqrt{2\mathsf{KL}(P_1||P_2)}. \qquad \text{(Pinsker)}$$

So we may find $\hat{P}$ that minimizes $\mathsf{KL}(P^*||\hat{P})$:

$$\hat{P} \leftarrow \operatorname*{argmin}_{P\in\mathcal{M}} \sum_{x\in\mathcal{X}} P^*(x) \log \frac{P^*(x)}{P(x)}$$

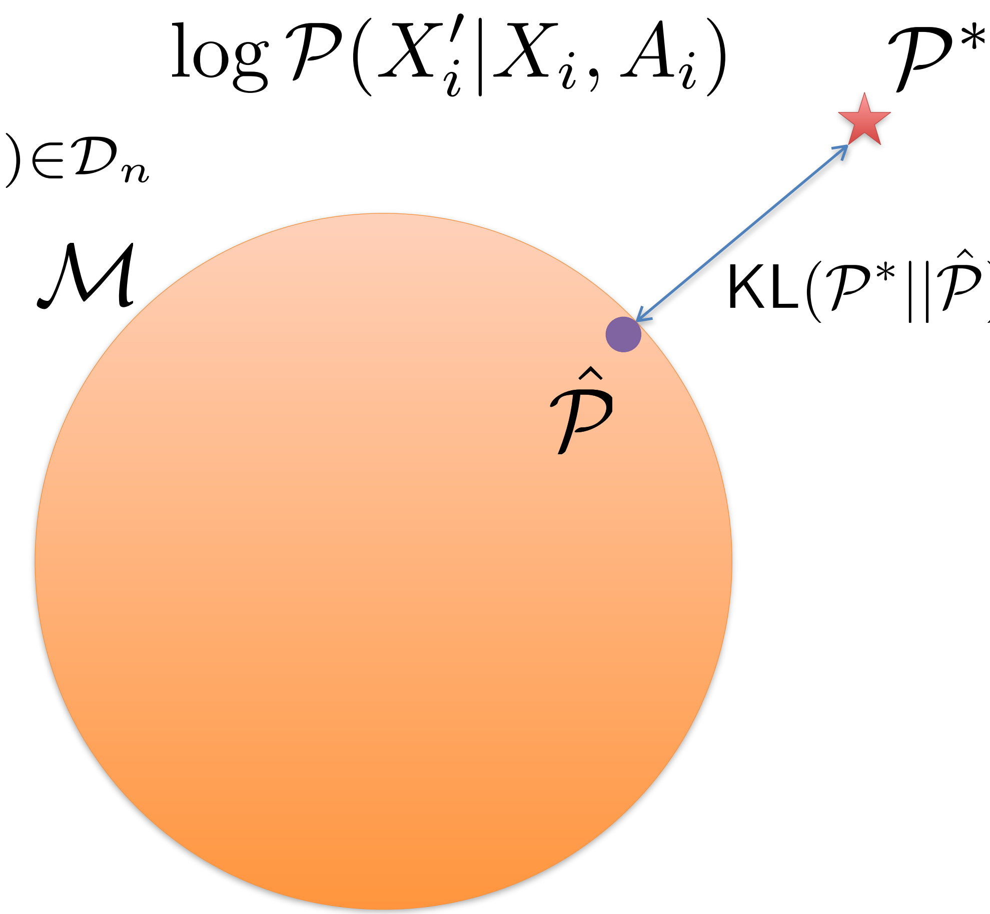
Or its empirical version: Given $\mathcal{D}_n = \{X_i\}_{i=1}^n$ with $X_i \sim P^*$, define the empirical measure $P_n^*(\cdot) = \frac{1}{n}\sum_{i=1}^n \delta_{X_i}(\cdot)$.
The Maximum Likelihood Estimator (MLE) is

$$\hat{P} \leftarrow \operatorname*{argmin}_{P\in\mathcal{M}} \mathsf{KL}(P_n^*||P) \equiv \operatorname*{argmax}_{P\in\mathcal{M}} \frac{1}{n} \sum_{X_i\in\mathcal{D}_n} \log P(X_i).$$

VAML vs. MLE

$$\left|\left\langle \mathcal{P}^*(\cdot|x,a) - \hat{\mathcal{P}}(\cdot|x,a)\,,\, V \right\rangle\right| \le \underbrace{\left\|\mathcal{P}^*(\cdot|x,a) - \hat{\mathcal{P}}(\cdot|x,a)\right\|_1}_{\le \sqrt{2\mathsf{KL}\left(\mathcal{P}^*(\cdot|x,a) || \hat{\mathcal{P}}(\cdot|x,a)\right)}} \|V\|_\infty$$

$$\hat{\mathcal{P}} \leftarrow \operatorname*{argmin}_{\mathcal{P}\in\mathcal{M}} \mathsf{KL}(\mathcal{P}_n^* || \mathcal{P}) = \operatorname*{argmax}_{\mathcal{P}\in\mathcal{M}} \frac{1}{n} \sum_{(X_i, A_i, X_i') \in \mathcal{D}_n} \log \mathcal{P}(X_i' | X_i, A_i)$$

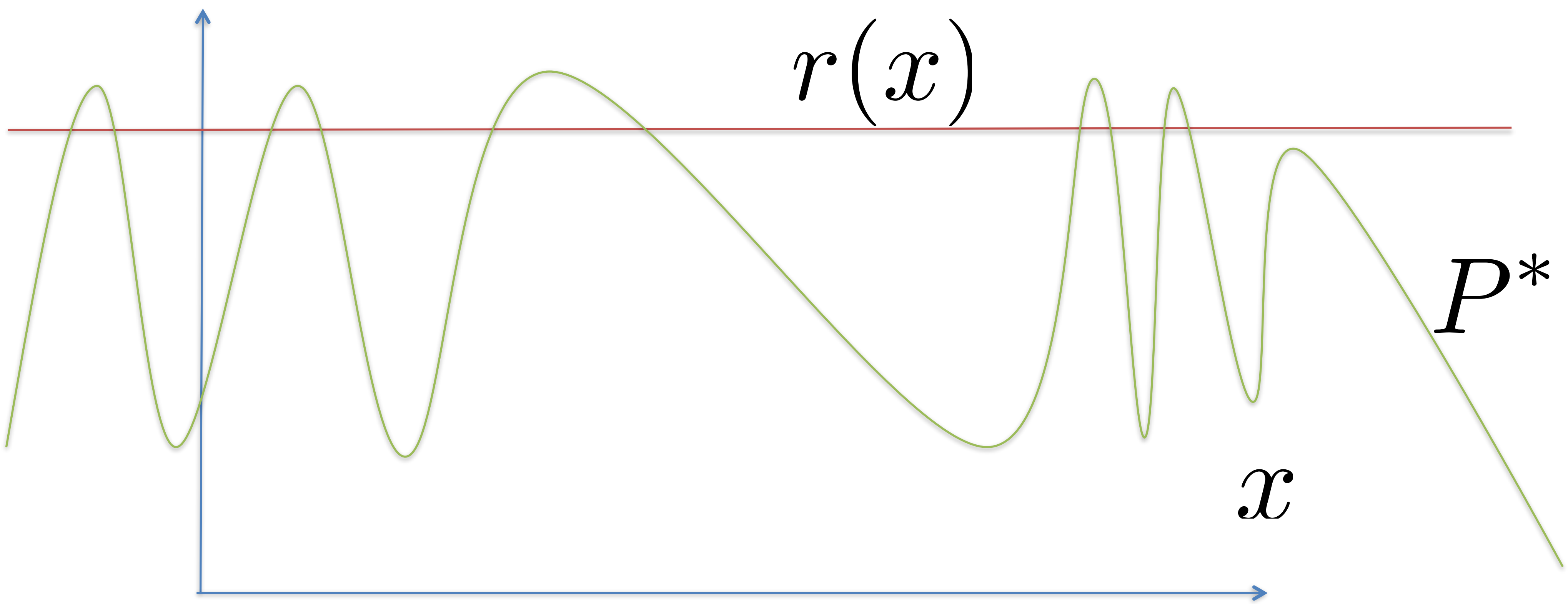


MLE ignores any possible information about the decision problem.

Joseph, Geramifard, Roberts, How, Roy, ICRA, 2013 — Silver, van Hasselt, Hessel, et al., ICML, 2017 — Farquhar, Rocktaeschel Igl, Whiteson, ICLR, 2018 — Oh, Singh, Lee, NIPS, 2017

$$\hat{P}r \approx P^* r$$

$$\text{i.e.,} \int \hat{P}(\mathrm{d}x')r(x') \approx \int P^*(\mathrm{d}x')r(x')$$



- No need to accurately (in the KL sense) estimate the true model.
- Any model is sufficient.
- MLE is an overkill for this reward (value) function.

$$c^2(\hat{\mathcal{P}}, \mathcal{P}^*; V)(x,a) = \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right|^2$$

$$\Big\downarrow \text{ Pointwise to expectation}$$

$$c^2_{2,\nu}(\hat{\mathcal{P}}, \mathcal{P}^*; V) = \int \mathrm{d}\nu(x,a) \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right|^2$$

$$c^2_{2,\nu}(\hat{\mathcal{P}}, \mathcal{P}^*; V) = \int \mathrm{d}\nu(x,a) \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right|^2$$

Unknown!

- **Value-Aware Model Learning (VAML):** Suppose that Planner uses a value function space $\mathcal{F}$ to represent the value function. We learn a model in $\mathcal{M}$ that is uniformly good for any function in $\mathcal{F}$.

- **Iterative VAML:** Learn models by benefiting from how Approximate Value Iteration (AVI)-based Planner generates value functions and uses models.

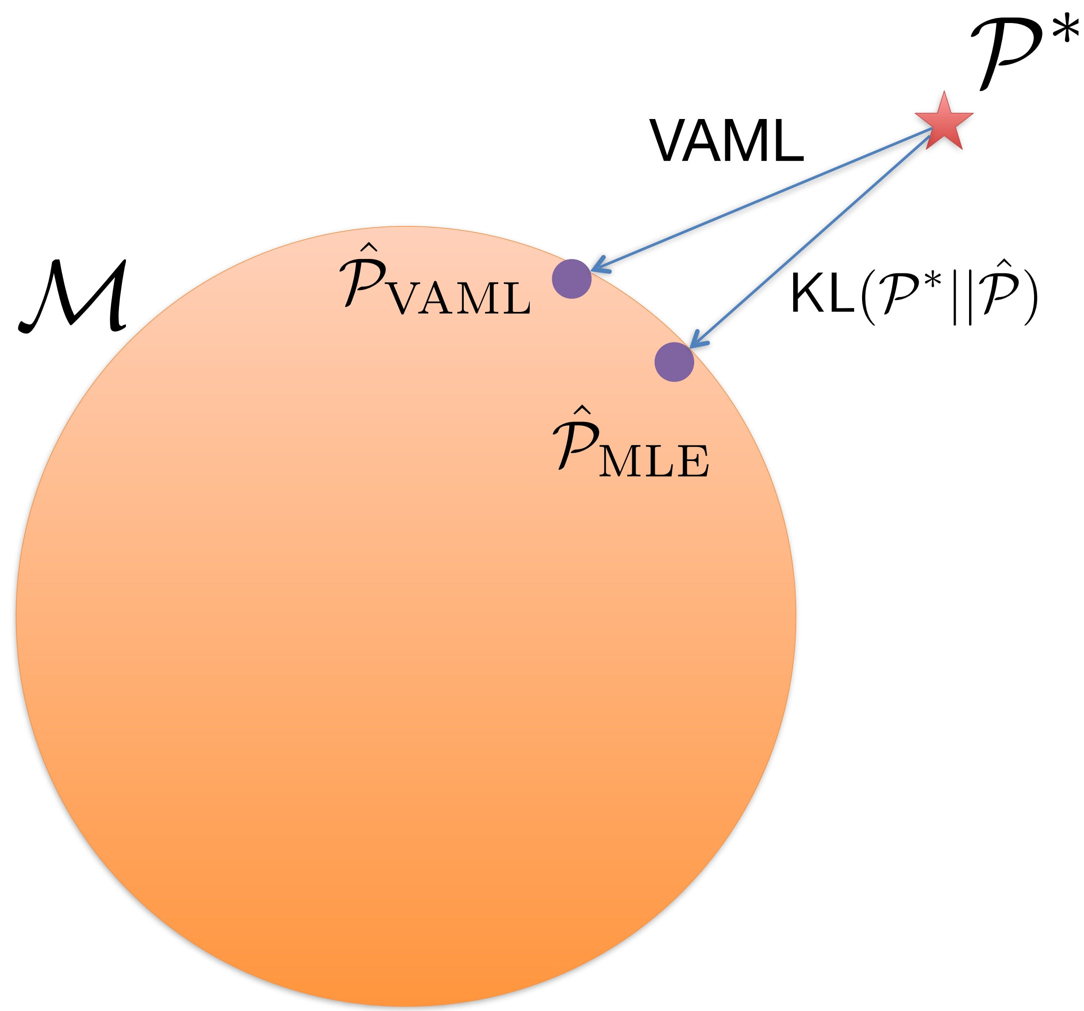
$$c_{2,\nu}^2(\hat{\mathcal{P}}, \mathcal{P}^*; V) = \int \mathrm{d}\nu(x,a) \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right|^2$$

Unknown!

Suppose that Planner uses a value function space $\mathcal{F}$ to represent the value function. We learn a model in $\mathcal{M}$ that is uniformly good for any function in $\mathcal{F}$.

$$c_{2,\nu}^2(\hat{\mathcal{P}}, \mathcal{P}^*) = \int \mathrm{d}\nu(x,a) \sup_{V \in \mathcal{F}} \left| \int \left[\mathcal{P}^*(\mathrm{d}x'|x,a) - \hat{\mathcal{P}}(\mathrm{d}x'|x,a) \right] V(x') \right|^2$$

VAML vs. MLE



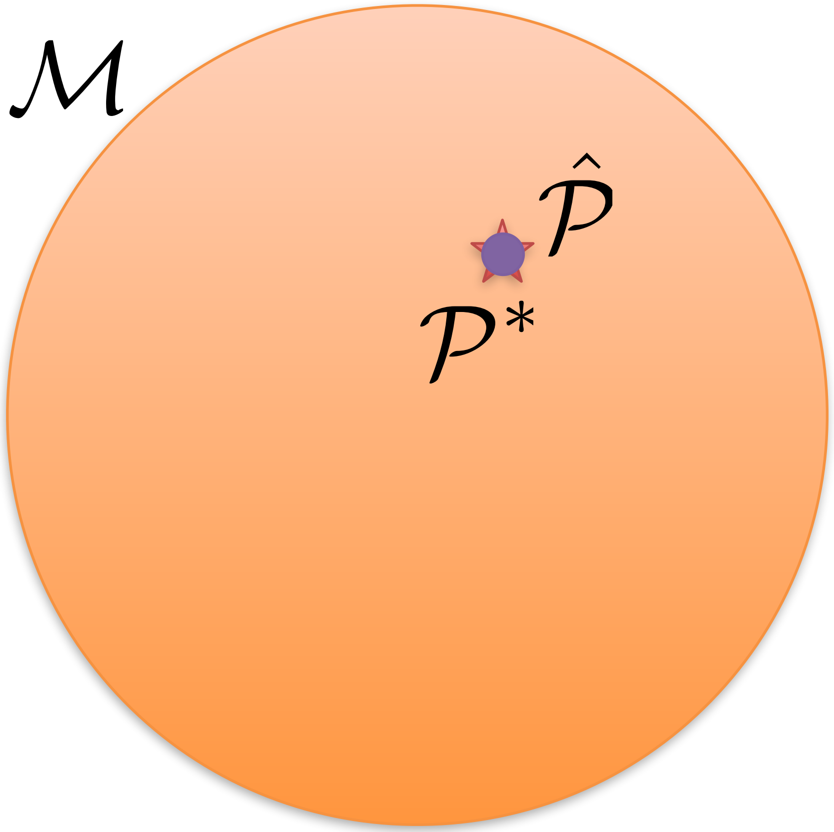
$\mathcal{P}^*$
VAML
$\mathcal{M}$
$\hat{\mathcal{P}}_{\mathrm{VAML}}$
KL$(\mathcal{P}^*||\hat{\mathcal{P}})$
$\hat{\mathcal{P}}_{\mathrm{MLE}}$

VAML vs. MLE: Mismatched Model Class

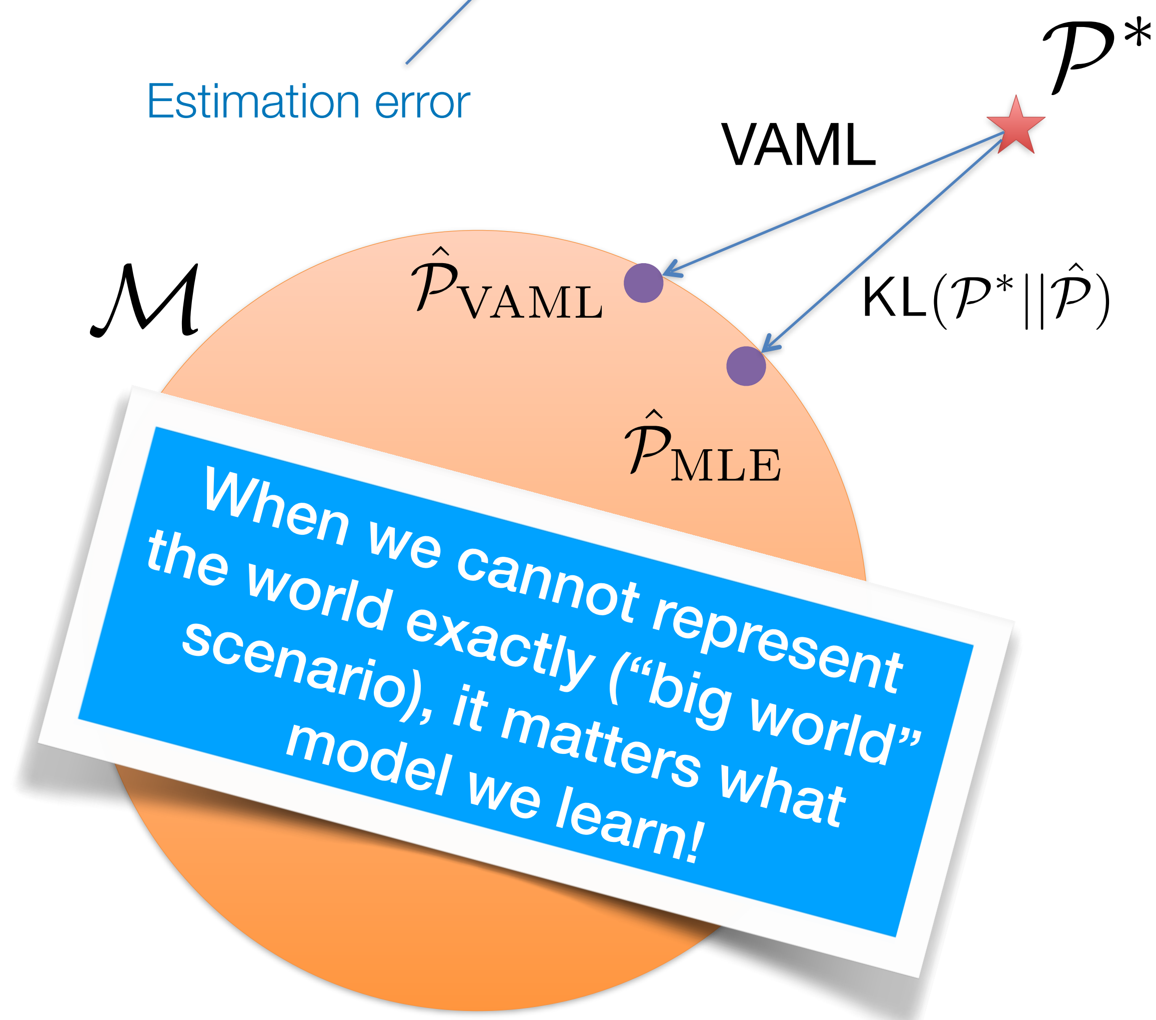
$$\mathbb{E}\left[\sup_{V\in\mathcal{F}}\left|(\hat{\mathcal{P}}_Z-\mathcal{P}^*_Z)V\right|^2\right] \leq \inf_{\mathcal{P}\in\mathcal{M}} \mathbb{E}\left[\sup_{V\in\mathcal{F}}\left|(\mathcal{P}_Z-\mathcal{P}^*_Z)V\right|^2\right] + O\left(B^\alpha\sqrt{\frac{\log(1/\delta)}{n}}\right)$$

Model (function) approximation error

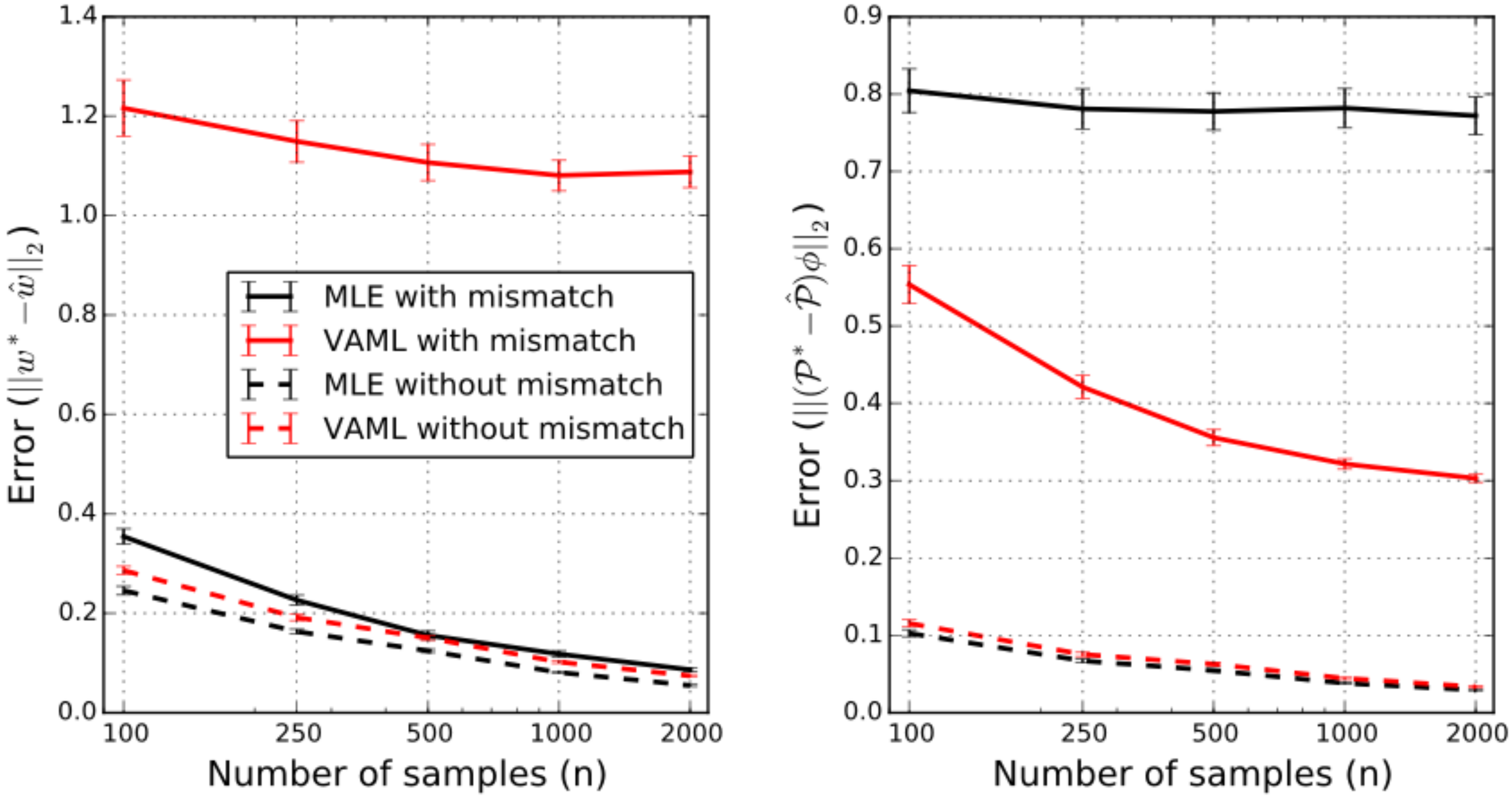
Estimation error



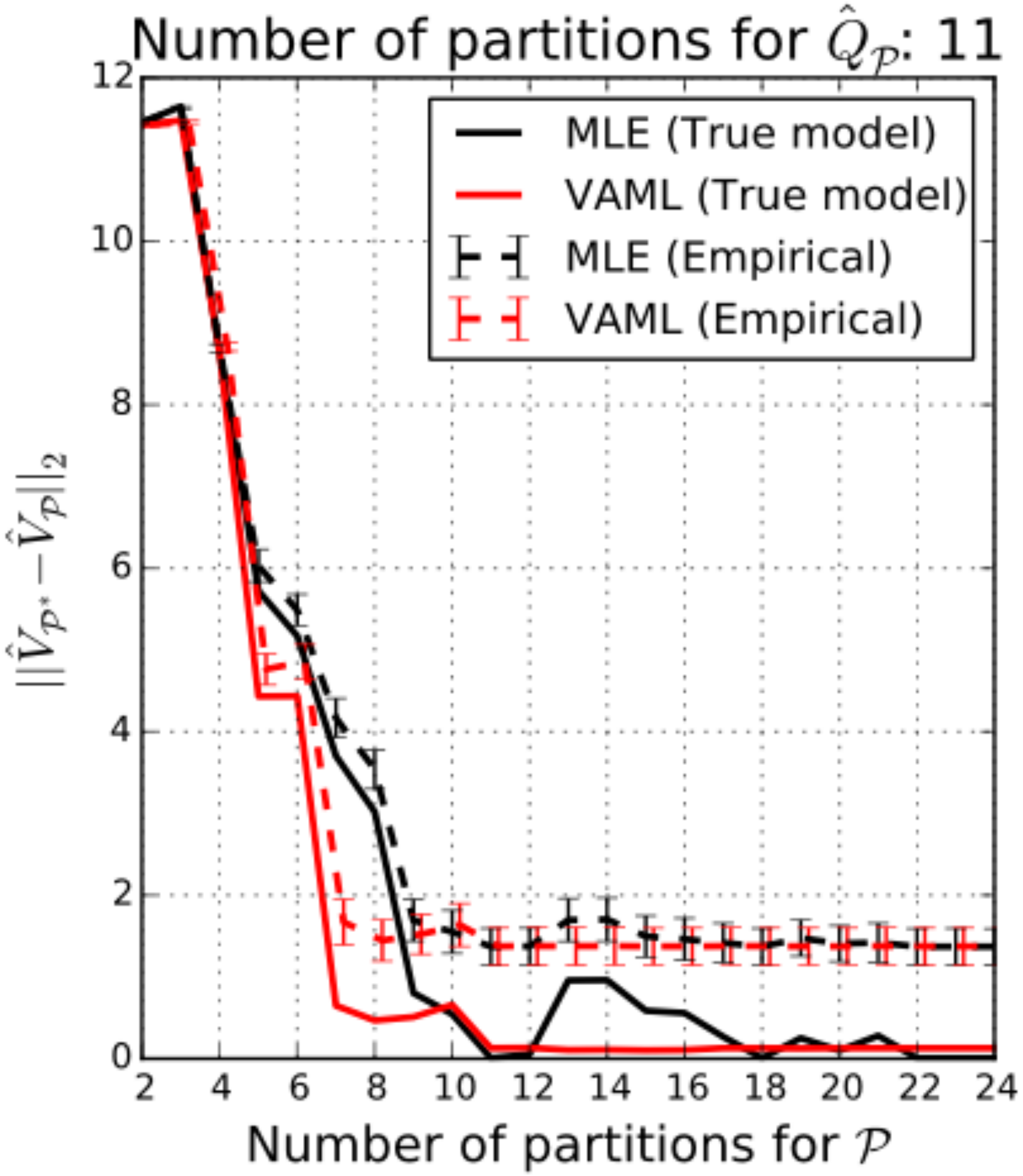
Matched

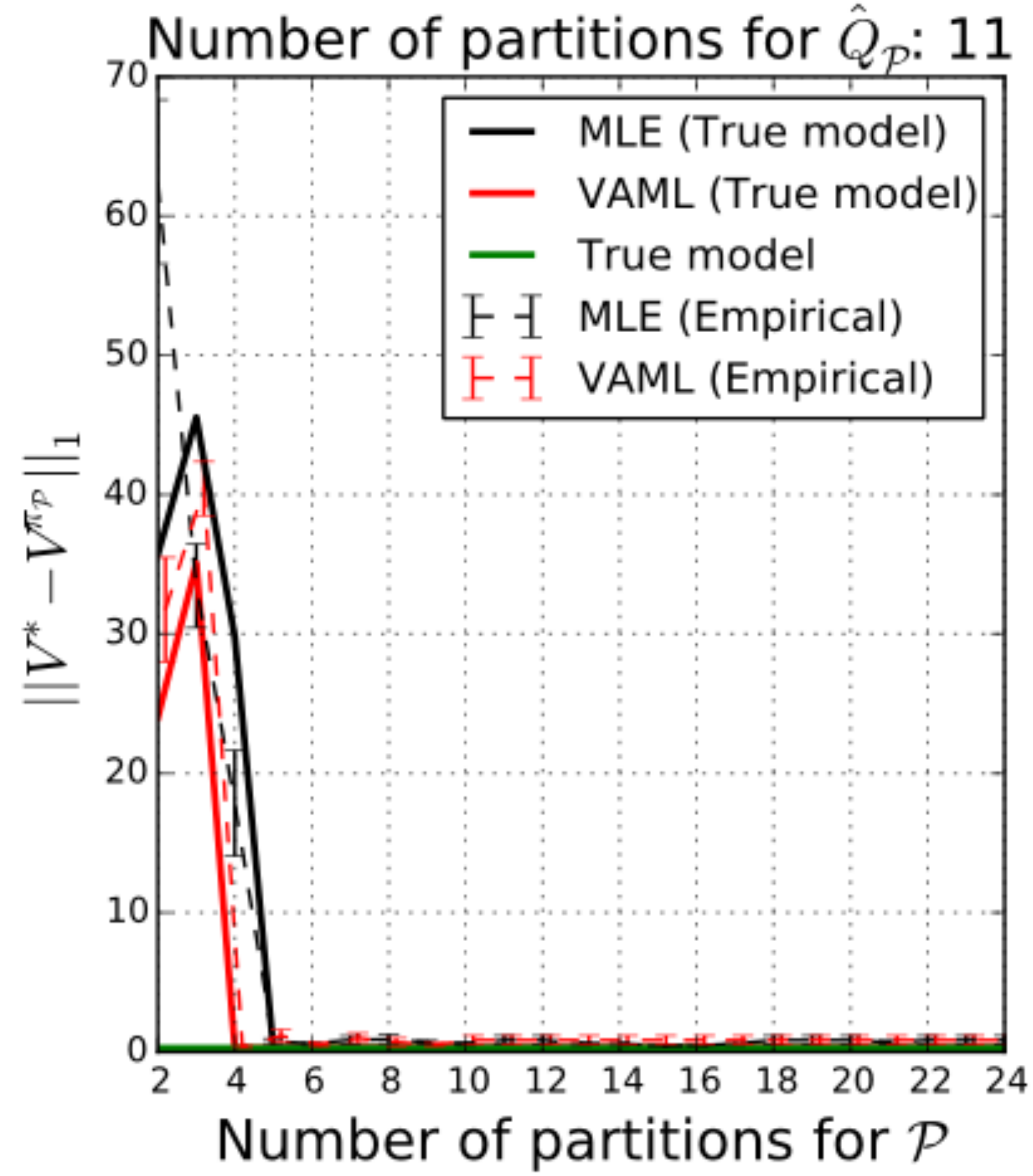


Mismatched



Domain: 10-dim Gaussian/Exponential
Model: Gaussian





Domain: Finite-state random walk MDP
Model: State aggregation

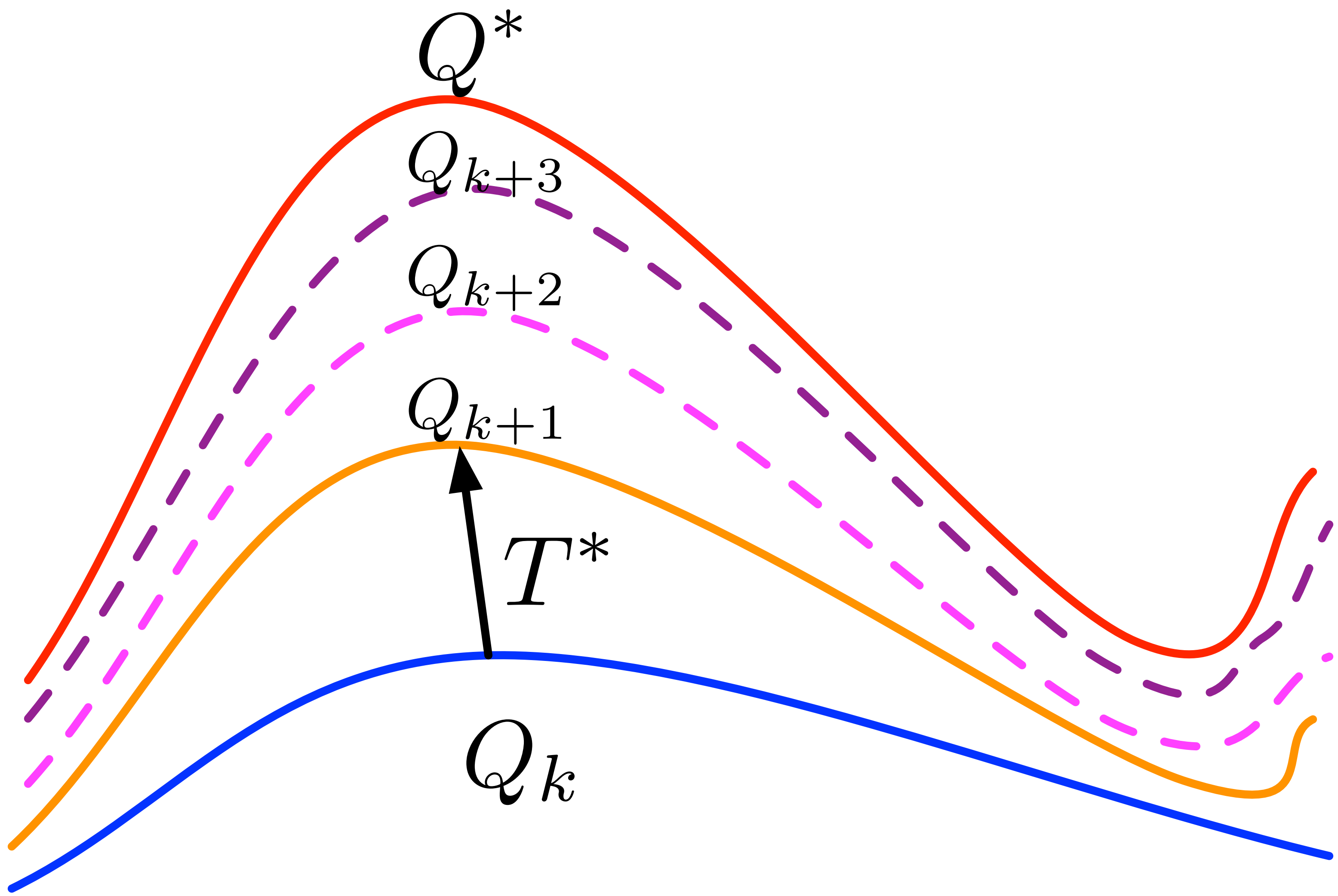
$$\hat{\mathcal{P}}_{\mathrm{VAML}} \leftarrow \operatorname*{argmin}_{\hat{\mathcal{P}} \in \mathcal{M}} \frac{1}{n} \sum_{(X_i, A_i, X_i') \in \mathcal{D}_n} \sup_{V \in \mathcal{F}} \left| V(X_i') - \int \hat{\mathcal{P}}(\mathrm{d}x' | X_i, A_i) V(x') \right|^2$$

Solving VAML optimization problem might be difficult for arbitrary function space!

Iterative VAML

AMF, "Iterative Value-Aware Model Learning," Neural Information Processing Systems (NeurIPS), 2018

Value Iteration



$$Q_{k+1} \leftarrow T^*_{\mathcal{P}^*} Q_k \triangleq r + \gamma \mathcal{P}^* V_k$$

$$V_k(x) \triangleq \max_a Q_k(x, a)$$

Iterative VAML

$$Q_0 \leftarrow r$$

$$Q_1 \leftarrow T^*_{\mathcal{P}^*} V_0 = r + \gamma \mathcal{P}^* V_0$$

$$Q_2 \leftarrow T^*_{\mathcal{P}^*} V_1 = r + \gamma \mathcal{P}^* V_1$$

$$\vdots$$

$$Q_{k+1} \leftarrow T^*_{\mathcal{P}^*} V_k = r + \gamma \mathcal{P}^* V_k$$

$$\hat{\mathcal{P}} V_0 = \mathcal{P}^* V_0$$

$$\hat{\mathcal{P}} V_1 = \mathcal{P}^* V_1$$

$$\hat{\mathcal{P}} V_k = \mathcal{P}^* V_k$$

$$\hat{\mathcal{P}} V_k \approx \mathcal{P}^* V_k$$

Iterative VAML

$$Q_0 \leftarrow r$$

$$Q_1 \leftarrow T^*_{\mathcal{P}^*} V_0 = r + \gamma \mathcal{P}^* r$$

$$Q_2 \leftarrow T^*_{\mathcal{P}^*} V_1 = r + \gamma \mathcal{P}^* V_1$$

$$\vdots$$

$$Q_{k+1} \leftarrow T^*_{\mathcal{P}^*} V_k = r + \gamma \mathcal{P}^* V_k$$

$$\boxed{\hat{\mathcal{P}} V_k \approx \mathcal{P}^* V_k}$$

$$\hat{\mathcal{P}}^{(k)} \leftarrow \underset{\mathcal{P} \in \mathcal{M}}{\operatorname{argmin}} \left\| (\mathcal{P} - \mathcal{P}^*) \hat{V}_k \right\|_2^2 = \int \left| (\mathcal{P} - \mathcal{P}^*)(\mathrm{d}x' | z) \max_{a'} \hat{Q}_k(x', a') \right|^2 \mathrm{d}\nu(z)$$

$$\hat{Q}_{k+1} \leftarrow T^*_{\hat{\mathcal{P}}^{(k)}} \hat{Q}_k$$

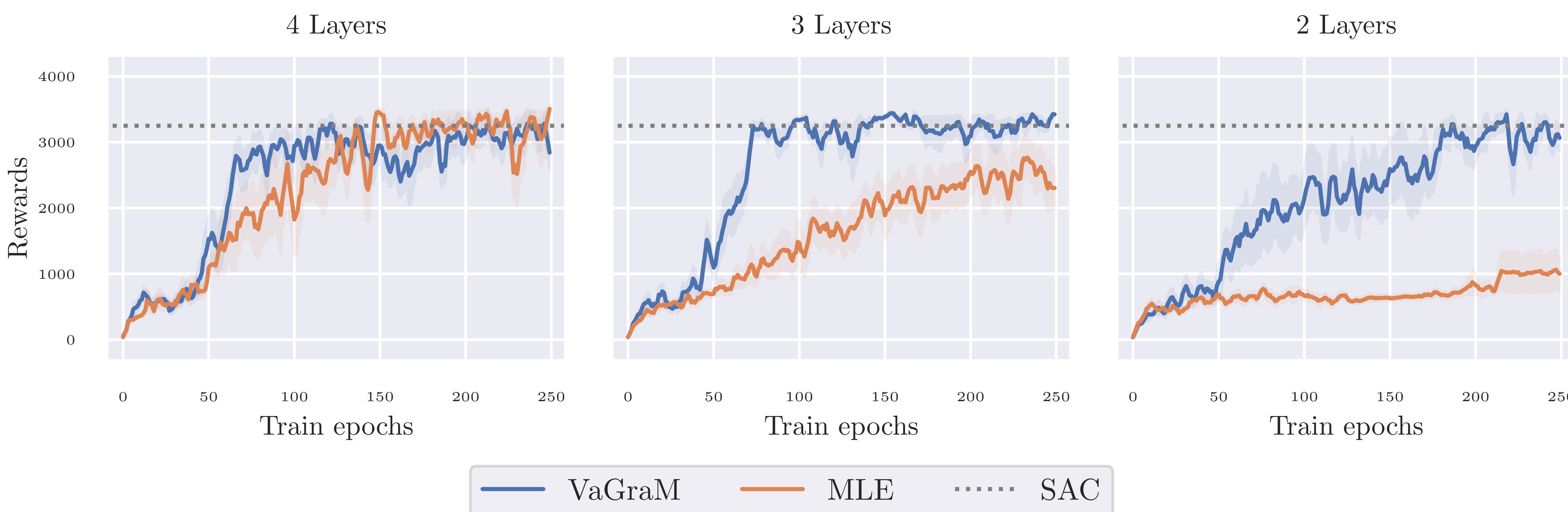
VAML with Deep Neural Networks

How to successfully implement VAML within NN?

VaGraM: Value-Gradient aware Model learning
(ICLR 2022)

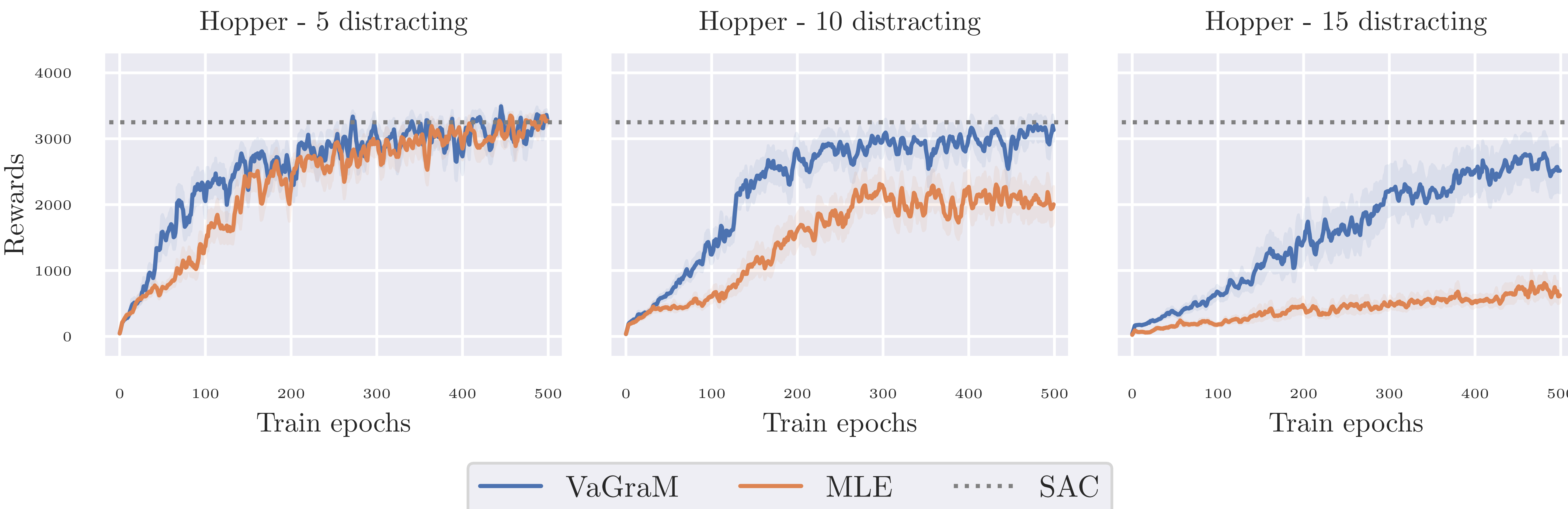
λ-AC: LAtent Model-Based Decision-Aware Actor-Critic
(arXiv, 2023)

VaGraM: Effect of Model Capacity

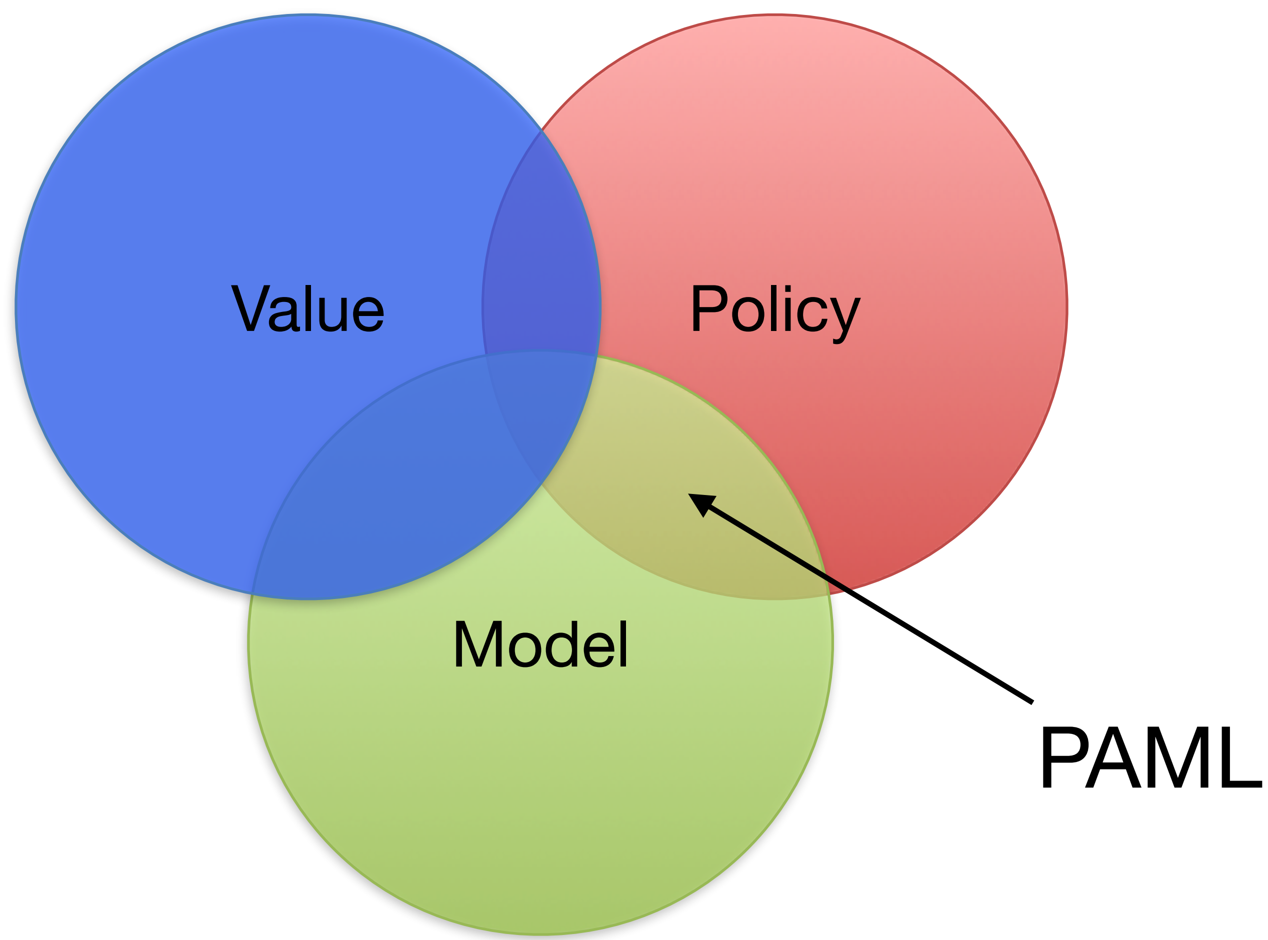


Hopper

VaGraM: Effect of Distracting Dimensions



Policy-Aware Model Learning (PAML)



Abachi, Ghavamzadeh, **AMF**, "Policy-Aware Model Learning for Policy Gradient Methods," [preprint](), 2020.

Policy Gradient

Policy parameterized by $\theta \in \Theta$.
Performance objective of an agent starting from an initial probability distribution $\rho \in \bar{\mathcal{M}}(\mathcal{X})$ and following policy π_θ in an MDP $\mathcal{P}$:

$$J_\rho(\pi_\theta; \mathcal{P}) \triangleq \int \mathrm{d}\rho(x) V_{\mathcal{P}}^{\pi_\theta}(x).$$

Policy Gradient:

$$\theta_{k+1} \leftarrow \theta_k + \eta \nabla_\theta J_\rho(\pi_{\theta_k}; \mathcal{P})$$

Policy Gradient

$$\nabla_\theta J(\pi_\theta) = \frac{\partial J(\pi_\theta)}{\partial \theta} = \sum_{k \geq 0} \gamma^k \int \mathrm{d}\rho(x) \int \mathcal{P}^{\pi_\theta}(\mathrm{d}x'|x;k) \sum_{a' \in \mathcal{A}} \frac{\partial \pi_\theta(a'|x')}{\partial \theta} Q^{\pi_\theta}(x', a')$$

$$= \frac{1}{1-\gamma} \int \rho_\gamma(\mathrm{d}x; \mathcal{P}^{\pi_\theta}) \sum_{a \in \mathcal{A}} \pi_\theta(a|x) \frac{\partial \log \pi_\theta(a|x)}{\partial \theta} Q^{\pi_\theta}(x, a).$$

$$\rho_\gamma^\pi(\cdot) = \rho_\gamma(\cdot; \mathcal{P}^\pi) \triangleq (1-\gamma) \sum_{k \geq 0} \gamma^k \int \mathrm{d}\rho(x) \mathcal{P}^\pi(\cdot|x;k).$$

Discounted future-state stationary distribution

Policy-Aware Model Learning

Goal:

Finding a model that computes the **Policy Gradient** as accurate as possible.

$$\nabla_\theta J_\rho(\pi_{\theta_k}; \hat{\mathcal{P}}) \approx \nabla_\theta J_\rho(\pi_{\theta_k}; \mathcal{P}^*)$$

PAML vs MLE

$$\left\| \nabla_\theta J(\pi_\theta) - \nabla_\theta \hat{J}(\pi_\theta) \right\|_p \leq \frac{\gamma}{(1-\gamma)^2} Q_{\max} B_p \times \begin{cases} c_{\mathrm{PG}}(\rho, \nu; \pi_\theta) \sqrt{2\mathsf{KL}_{1(\nu)}(\mathcal{P}^{\pi_\theta} \| \hat{\mathcal{P}}_{\pi_\theta})}, \\ 2\sqrt{2\mathsf{KL}_\infty(\mathcal{P}^{\pi_\theta} \| \hat{\mathcal{P}}_{\pi_\theta})}. \end{cases}$$

Minimized by PAML

Minimized by MLE

$$\pi_\theta(a|x) = \frac{\exp\left(\phi^\top(a|x)\theta\right)}{\int \exp\left(\phi^\top(a'|x)\theta\right) \mathrm{d}a'} \qquad c_{\mathrm{PG}}(\rho, \nu; \pi) \triangleq \left\| \frac{\mathrm{d}\rho_\gamma^\pi}{\mathrm{d}\nu} \right\|_\infty$$

$$\mathsf{KL}_\infty(\mathcal{P}_1^\pi \| \mathcal{P}_2^\pi) = \sup_{x \in \mathcal{X}} \mathsf{KL}(\mathcal{P}_1^\pi(\cdot|x) \| \mathcal{P}_2^\pi(\cdot|x)), \quad \mathsf{KL}_{1(\nu)}(\mathcal{P}_1^\pi \| \mathcal{P}_2^\pi) = \int \mathrm{d}\nu(x) \mathsf{KL}(\mathcal{P}_1^\pi(\cdot|x) \| \mathcal{P}_2^\pi(\cdot|x)).$$

Integral Probability Metric & Model Learning

Given two probability distributions $\mu_1, \mu_2 \in \bar{\mathcal{M}}(\mathcal{X})$ defined over the set $\mathcal{X}$ and a space of functions $\mathcal{F} : \mathcal{X} \to \mathbb{R}$, the Integral Probability Metric (IPM) distance is defined as $d_{\mathcal{F}}(\mu_1, \mu_2) = \sup_{f \in \mathcal{F}} \left| \int f(x) \left(\mathrm{d}\mu_1(x) - \mathrm{d}\mu_2(x) \right) \right|$.

- Total Variation distance: $\mathcal{F}$ is the space of bounded measurable function. (Also recall that $\|\mu_1 - \mu_2\|_{\mathrm{TV}} \le \sqrt{2\mathsf{KL}(\mu_1 || \mu_2)}$).
- 1-Wasserstein distance: $\mathcal{F}$ is the space of 1-Lipschitz functions. Special case of VAML (Asadi et al., 2018).
- VAML: $\mathcal{F}$ is the space of value functions.
- IterVAML: $\mathcal{F}$ is the most recent value function, i.e., $\mathcal{F} = \{V_k\}$.
- PAML:
    1. $\mathcal{F}$ has a single function $f(x) = \mathbb{E}_{A \sim \pi_\theta(\cdot|x)} \left[\nabla_\theta \log \pi_\theta(A|x) Q^{\pi_\theta}(x, A) \right]$.
    2. Comparison is not between $\mathcal{P}^*$ and $\hat{\mathcal{P}}$, but between $\rho_\gamma(\cdot; \mathcal{P}^{*\pi_\theta})$ and $\rho_\gamma(\cdot; \hat{\mathcal{P}}^{\pi_\theta})$.

IPM: Müller, 1997 — Asadi et al., 2018.

Other DAML Approaches

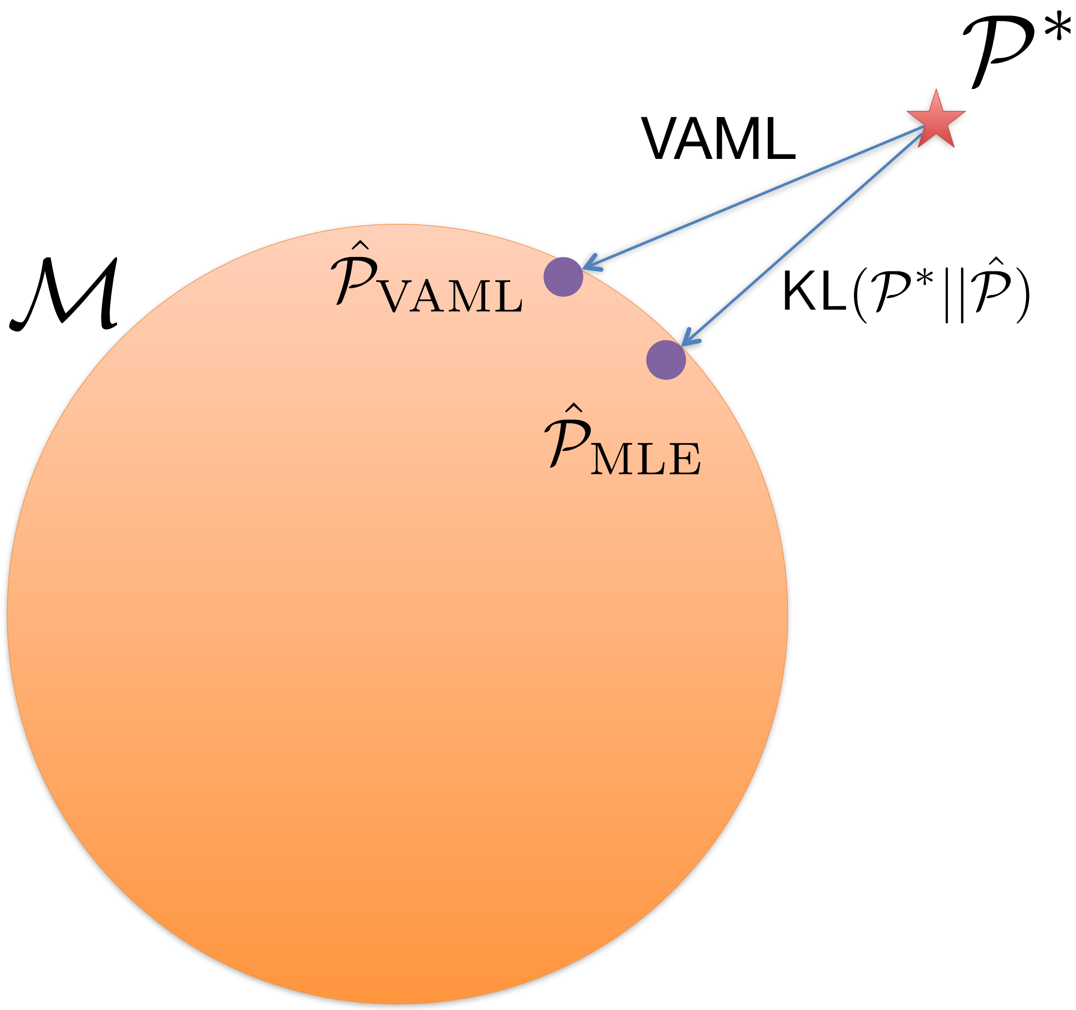
Several methods in the RL literature might be interpreted as doing some form of DAML, though sometimes it is not explicitly mentioned.
Some examples:

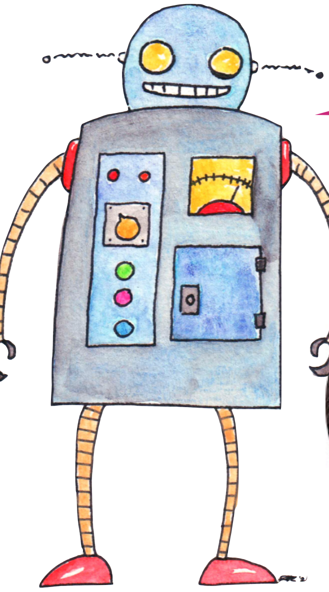
- Joseph et al., ICRA, 2013.
- Predictron (Silver et al., 2017)
- VPN (Oh et al., 2017)
- TreeQN (Farquhar et al., 2018)
- Gradient-Aware Model-based Policy Search (D'Oro et al., 2020)
- muZero (Schrittwieser et al., 2019)
- Value-targeted regression (Ayoub et al., 2020)
- Value equivalence viewpoint (Grimm et al., 2020)
- A few others in non-RL context (Tulabandhula and Rudin, 2013; Kao and Van Roy, 2014; Elmachtoub and Grigas, 2017, Donti et al., 2017)

Take-Home Message: Decision-Aware Model Learning

We should incorporate the structure of the **decision problem** and planner into model learning.

The world is too large to learn everything about it!





A subset of Adaptive Agents Lab (Adage)

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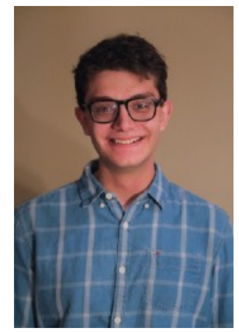
Arash Ahmadian (UG)



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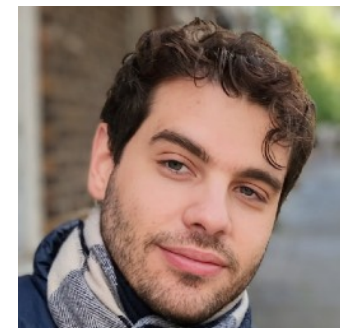

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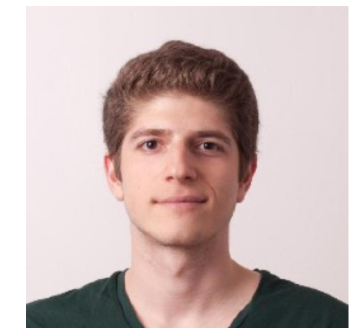

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Amin Rakhsha (PhD)

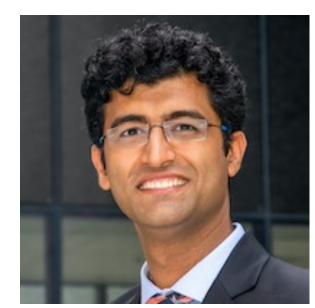
Dr. Claas Voelcker

Andrew Wang (UG)

...and friends

André Barreto

Murat Erdogdu



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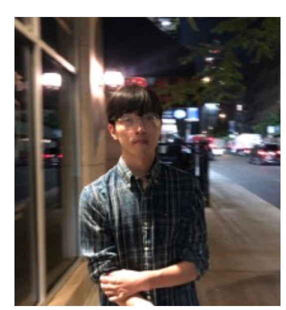

Jongmin Lee

Daniel Nikovski

Ernest Ryu

Do you want to join?!
I am recruiting!

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